Name: ________________________________ Student Number: ____________________	CISC 203 Discrete Mathematics for Computing Science Test 3, Fall 2011 Professor Mary McCollam

This test is 50 minutes long and there are 40 marks. **Please write in pen and only in the box marked "Answer".** This is a closed-book exam. No computers or calculators are allowed.

NOTES:

*** Justify your answers *** to all of the problems (give explanation or show work).

All solutions with factorials only need to be reduced to factorial form, e.g., $\frac{12!\ 5!}{2!\ 4!}$

Question 1: [10 marks]

a) How many people are needed to guarantee that at least two were born on the same day of the week and in the same month (perhaps in different years)?

Hint: Use the Pigeonhole principle.

Answer:

Using the pigeonhole principle, we need 12 • 7 + 1 = 85 people,

since there are 12 months and 7 days, so 12 • 7 month/day categories in which to distribute the people

b) How many bit strings of length 9 either start with 000 or end with 1111?

Answer:

$2^6 + 2^5 - 2^2$

This is the number of bit strings of length 9 that start with 000 + the number that end with 1111 minus the number that both start with 000 and end with 1111

Thus we're using the inclusion-exclusion principle with the sum rule

Question 2: [10 marks] By algebraic manipulation, show that if n is a positive integer, then $C(2n, 2) = 2 \times C(n, 2) + n^2$

Answer:

$$C(2n, 2) = \frac{(2n)!}{2!\,(2n-2)!} = \frac{(2n)(2n-1)(2n-2)\ldots(2)(1)}{(2)\;(2n-2)(2n-3)\ldots.(2)(1)} = \frac{(2n)(2n-1)}{2} = n(2n-1)$$

$$2C(n, 2) + n^2 = \frac{(2)n!}{2!\,(n-2)!} + n^2 = \frac{(2)(n)(n-1)(n-2)\ldots(1)}{(2)(n-2)(n-3)\ldots(1)} + n^2 = n(n-1) + n^2 = n(2n-1)$$

Question 3: [10 marks]

a) If there are **seven women** and **nine men** on the faculty in the mathematics department at a school, how many ways are there to select a committee of **four** members of the department if **at least one man** must be on the committee?

Answer:

There are C(16, 4) ways to select a committee if there are no restrictions.

There are C(7, 4) ways to select a committee from just the 7 women.

Therefore there are C(16, 4) – C(7, 4) $= \dfrac{16!}{4!\,12!} - \dfrac{7!}{4!\,3!}$

ways to select a committee with at least one woman.

b) If we have a standard deck of **52** playing cards, and we are dealing **5** cards to a player in a poker game, how many ways are there to deal a **5-card hand** that consists of a **full house**?

A full house means that there are two matching cards of one value (a pair) and three matching cards of a different value, for example: 4♥, 4♣, 5♠, 5♦, 5♣

Answer:

Choose the value of the pair: C(13,1)

Choose the suit of the pair: C(4,2)

Choose the value of the triple from the remaining 12 values: C(12,1)

Choose the suit of the triple: C(4,3)

Total: C(13,1) • C(4,2) • C(12,1) • C(4,3) = 13 • 4!/(2!2!) • 12 • 4

Question 4: [10 marks]

a) A croissant shop has plain bagels, onion bagels, poppy seed bagels, pumpernickel bagels, egg bagels, sesame seed bagels, and raisin bagels. How many ways are there to choose **two dozen** bagels with at least **two of each kind**?

Answer:

First, pick the two of each kind. This leaves 10 left to pick without restriction and with repetitions allowed, which is

$$C(7 + 10 - 1, 10) = C(16, 10)$$

$$= \frac{16!}{10!\,6!}$$

b) How many different strings of length **14** can be made that contain **3** As, **4** Es, **2** Ms, **2** Os, **1** P, and **2** Ts?

Answer:

$$\frac{14!}{3!\,4!\,2!\,2!\,1!\,2!}$$, since this is a problem of permutations with repetitions