1. Consider the following languages. For each, show whether or not Rice’s theorem shows that the language is undecidable.
   (a) \( A = \{ \langle M \rangle \mid M \text{ writes a blank symbol on the input portion of the tape} \} \)
   (b) \( B = \{ \langle M \rangle \mid L(M) \text{ is recognizable} \} \)
   (c) \( C = \{ \langle M \rangle \mid L(M) \text{ is finite} \} \)
   (d) \( D = \{ \langle M \rangle \mid A_{TM} \leq_m L(M) \} \)

2. Let \( \Sigma \) be a finite alphabet. We denote by \( FIN(\Sigma) \) the set of finite languages over \( \Sigma \). That is,
   \[
   FIN(\Sigma) = \{ L \subseteq \Sigma^* \mid L \text{ is finite} \}.
   \]
   Show that \( FIN(\Sigma) \) is countable.

3. A **useless state** is a state that is never entered on any input string. Consider the problem of determining whether a Turing machine has any useless states. Formulate this problem as a language and show it is undecidable.

4. Show that the Post Correspondence Problem is decidable over a unary alphabet.

5. Let \( B \) be a language over \( \Sigma \). Are the following true or false? Justify your answers.
   (a) If \( B \leq_m A_{TM} \), then \( B \) is decidable.
   (b) If \( B \leq_m A_{TM} \), then \( B \) is recognizable.

6. **Bonus question.** A **homomorphism** is a function \( \varphi : \Sigma^* \to \Delta^* \) where \( \Sigma \) and \( \Delta \) are alphabets. Let \( w = a_1a_2\cdots a_n \in \Sigma^* \) and \( L \subseteq \Sigma^* \) be a language. Then
   \[
   \varphi(w) = \varphi(a_1)\varphi(a_2)\cdots \varphi(a_n)
   \]
   and
   \[
   \varphi(L) = \{ \varphi(w) \mid w \in L \}.
   \]
   Show that the class of decidable languages is not closed under homomorphism.