Solid Hypercodes
UWORCS 2015

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What is a code?

```javascript
if(top !== self) {
  function calcWidth() {
    var wW = 0;
    if (typeof window.innerWidth == 'number') {
      wW = window.innerWidth;
    } else if (document.documentElement && document.documentElement.clientWidth > 0) {
      wW = document.documentElement.clientWidth;
    } else if (document.body && document.body.clientWidth > 0) {
      wW = document.body.clientWidth;
    }
    if (sH = window.innerHeight || document.documentElement.clientHeight || document.body.clientHeight) {
      wW = Math.max(wW, sH > wW ? sH : wW);
    }
  }
  calcWidth();
}
```
What is a code?

Terminology

0 symbol
What is a code?

Terminology

00110101 word \( (w) \)
What is a code?

Terminology

00110101 language ($L$)
10100101
00000011
10010110

:
What is a code?

Definition (Code)

A language $L$ is a code if every word $w$ has a unique $L$-factorization, that is, $u_1u_2\ldots u_m = v_1v_2\ldots v_n$ with $u_i, v_j \in L$ for all $i$ and $j$ implies $m = n$ and $u_i = v_i$ for all $i$. 
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Simpler Definition (Code)
A language $L$ is a code if every string composed of words from $L$ has a single decomposition.

Example
$L = \{0, 1\}$
$w = 00110101$
The information processing and transmission model:

All communication is represented by these five components.
For any message $w$, it is expected that $\delta(\gamma(w)) = w$.
However, the presence of noise may introduce errors.
The development of communication methods that ensure less noise, and thus a more reliable transmission, lies at the foundation of channel coding.

Classes of codes

- Prefix codes
- Suffix codes
- Infix codes
- Bifix codes
- Overlap-free codes
- Solid codes
- Hypercodes
- …
Definition (Solid code)

A language $L$ is a solid code if it satisfies the following conditions:

1. no word in $L$ is a subword of another word in $L$ (infix-freeness)
2. no proper prefix of a word in $L$ is a proper suffix of a word in $L$ (overlap-freeness)

Example

$L = \{00111, 01\}$

Does not meet infix-free property.

$L = \{0011, 1001\}$

Does not meet overlap-free property.

$L = \{00111, 010111\}$

This is a solid code!
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Example
$L = \{010, 0001000\}$
The first word is a proper subword of the second word.
Definition (Hypercode)
A language $L$ is a hypercode if no word in $L$ is a proper subword of another word in $L$.

Example
$L = \{010, 0001000\}$
The first word is a proper subword of the second word.

$L = \{1010000, 11100\}$
This is a hypercode!
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Definition (Solid hypercode)

A language $L$ is a solid hypercode if it is both a solid code and a hypercode.

By using solid hypercodes, transmitted words become resistant to errors both between code words and within code words.
Properties

The **solid** aspect allows for synchronization capabilities.
The **hyper** aspect allows for thorough protection against errors.
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**Checking for the solid hypercode property**

1a. Check if the language is a block code.
   - If so, then the language is hyper
   - Otherwise, continue
Properties

The **solid** aspect allows for synchronization capabilities.
The **hyper** aspect allows for thorough protection against errors.

Checking for the solid hypercode property

1a. Check if the language is a block code.
   ▶ If so, then the language is hyper
   ▶ Otherwise, continue

1b. Check the embedding order of each pair of words.
   ▶ If one word can be embedded in the other, then stop
   ▶ Otherwise, check the next pair of words
Properties

The **solid** aspect allows for synchronization capabilities. The **hyper** aspect allows for thorough protection against errors.

**Checking for the solid hypercode property**

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   ▶ If so, then the language is hyper
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1b. Check the embedding order of each pair of words.
   ▶ If one word can be embedded in the other, then stop
   ▶ Otherwise, check the next pair of words

2. Check the first symbol of each word.
   ▶ If symbols do not match, then stop
   ▶ Otherwise, check each pair of words for overlaps
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Overview

Diplomarbeit.cpp: a method of generating solid hypercodes [1]

- Functional, but very slow
- Short-term goal: to understand and improve this software tool
- Long-term goal: to generate large sets of solid hypercodes
- Problem: everything is written in German!
A detailed analysis of the most important parts of the original work allows non-German speakers to understand the author’s intent.

Two algorithms are discussed in the original work.

- Algorithm I, which enumerates all words and performs a check on each
  - Suffers from severe performance issues
- Algorithm II, which uses backtracking to enumerate possible words and “take a step back” if a check fails
  - Slightly better, but not yet perfect
Improvements

Performance

- modified `Next()` and `Previous()` methods to reduce redundant computation
- enhanced user input/output capabilities
- wrote new program to scan output file and keep “interesting” sets of code words

Cosmetic

- translated all variable names and comments to German
- added additional comments
- brought consistency to formatting and spacing
Improvements

Comparative run times of Diplomarbeit.cpp (in seconds)

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<th>Input</th>
<th>Original</th>
<th>Modified</th>
</tr>
</thead>
<tbody>
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<tr>
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<td>1052</td>
<td>1013</td>
</tr>
</tbody>
</table>
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## Conclusions
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- One of the most important aspects of communication is resilience against errors
- Solid hypercodes are resilient against errors
- An efficient method of checking the solid hypercode property was developed
- A method of generating sets of solid hypercodes was analyzed
- This method was modified for greater performance
- There is still room for improvement!
Future work

- How can we implement this checking method in the real world?
- How can we check non-binary alphabets?
- Will the generation method run faster using this check?
- Are there any other redundancies in the generation method?
References
