Module 7: Project Scheduling (PERT/CPM)
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Project

- A project is
  - a temporary endeavour undertaken to create a "unique" product or service

- A project is composed of
  - a number of related activities that are directed to the accomplishment of a desired objective

- A project starts when
  - at least one of its activities is ready to start

- A project is completed when
  - all of its activities have been completed
Key Concepts

- Triple Constraints
- Funnel Of Uncertainty
Activity

An activity

- Must have a clear start and a clear stop
- Must have a duration that can be forecasted
- May require the completion of other activities before it begins
- Should have some ‘deliverables’ for ease of monitoring
A project plan is a schedule of activities indicating:

- The start and stop for each activity. The start and stop of each activity should be visible and easy to measure.
- When a resource is required.
- Amount of required project resources.
Project Planning

- Managers should consider:
  - Resource availability
  - Resource allocation
  - Staff responsibility
  - Cash flow forecasting

- Managers need to monitor and re-plan as the project progresses towards its pre-defined goal
Work Breakdown Structure (WBS)

- Contains a list of activities for a project derived from:
  - Previous experience
  - Expert brainstorming

- WBS helps in:
  - Identifying the main activities
  - Breaking each main activity down into sub-activities which can further be broken down into lower level sub-activities

- WBS problems:
  - Too many levels
  - Too few levels
Creating WBS

- Phase based approach
- Product based approach
- Hybrid approach
Example of Phase-based Approach

Work Breakdown Structure (an extract)
Phase-based Approach

■ Advantage
  – Activity list likely complete and non-overlapping
  – WBS gives a structure that can be
    • refined as the project proceeds
    • used for determining dependencies among activities

■ Disadvantage
  – May miss some activities related to final product
Product based approach

- Product Breakdown Structure (PBS)
  - Shows how a system can be broken down into different products for development

A Product Breakdown Structure (an extract)
Hybrid Approach

- A mix of the phase-based and product-based approaches (most commonly used)
- The WBS consists of
  - a list of the products of the project; and
  - a list of phases for each product
IBM MITP (Managing the Implementation of Total Project)

IBM MITP is 5 levels:

- Level 1: Project
- Level 2: Deliverables (software, manuals etc)
- Level 3: Components: key work items that lead to the production of the deliverables
- Level 4: Work-packages: major work items or collection of related activities to produce a component (phases)
- Level 5: Tasks/activities (individual responsibility)
Project Scheduling

- **Steps**
  - Define activities
  - Sequence activities
  - Estimate time
  - Develop schedule

- **Techniques**
  - Gantt chart
  - CPM
  - PERT
  - Microsoft Project
Gantt Chart

- Developed in 1918 by H.L. Gantt
- Graph or bar chart with a bar for each project activity that shows passage of time
- Provides visual display of project schedule
- Limitations
  - Does not clearly indicate details regarding the progress of activities
  - Does not give a clear indication of interrelation between the activities
Example of Gantt Chart

- Design house and obtain financing: Months 1-3
- Lay foundation: Months 4-5
- Order and receive materials: Months 4-5
- Build house: Months 5-8
- Select paint: Months 5-7
- Select carpet: Months 7-9
- Finish work: Months 8-9
PERT/CPM

- **PERT (Program Evaluation and Review Technique)**
  - Developed by U.S. Navy for Polaris missile project
  - Developed for R&D projects where activity times are generally uncertain

- **CPM (Critical Path Method)**
  - Developed by DuPont & Remington Rand
  - Developed for industrial projects where activity times are generally known
CPM and PERT have been used to plan, schedule, and control a wide variety of projects:

- R&D of new products and processes
- Construction of buildings and highways
- Maintenance of large and complex equipment
- Design and installation of new systems
Program Evaluation and Review Technique (PERT)

- Primary objectives:
  - Shortest possible time
  - Coping with uncertain activity completion times, e.g.:
    • For a particular activity
    • The most likely completion time is 4 weeks but
    • It could be anywhere between 3 weeks and 8 weeks

- Developed by the US Navy for the planning and control of the Polaris missile program
Critical Path Method (CPM)

- Primary objectives:
  - Plan for the fastest completion of the project
  - Identify activities whose delays is likely to affect the completion date for the whole project
  - Very useful for repetitive activities with well known completion time

- Developed by Du Pont Chemical Company and published in 1958
  - Can we decrease the completion time by spending more money
CPM Calculation

- The forward pass
  - calculate the **earliest** start dates of the activities
    - to calculate the project completion date

- The backward pass
  - calculate the **latest** start dates for activities
    - to identify the critical path from the graph
Critical Path and Events

- Critical event: an event that has zero slack
- Critical path: a path joining critical events
- Benefit of Critical Path Analysis:
  - During planning stage
    - Shortening the critical path will reduce the overall project duration
  - During management stage
    - Pay more attention to those activities which fall in the critical path
Activity Float

- Time allowed for an activity to delay

- 3 different types:
  - **Total float** (without affecting project completion)
    = latest start date – earliest start date
  - **Free float** (without affecting the next activity)
    = earliest start date of next activity – latest end date of previous activity
  - **Interfering float** (= total float - free float)
Scheduling Network for House Building Project

1. Start
   - Design house and obtain financing

2. Lay foundations

3. Order and receive materials

4. Build house

5. Select paint

6. Select carpet

7. Finish work
Critical Path

A: 1-2-4-7
   3 + 2 + 3 + 1 = 9 months

B: 1-2-5-6-7
   3 + 2 + 1 + 1 + 1 = 8 months

C: 1-3-4-7
   3 + 1 + 3 + 1 = 8 months

D: 1-3-5-6-7
   3 + 1 + 1 + 1 + 1 = 7 months

- Critical path
  - Longest path through a network
  - Minimum project completion time
Activity Start Times

Start at 3 months

Start at 5 months

Finish at 9 months

Start at 6 months
Mode Configuration

- Activity number
- Activity duration
- Earliest start
- Earliest finish
- Latest start
- Latest finish

Diagram:

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Forward Pass

- Start at the beginning of CPM/PERT network to determine the earliest activity times

- Earliest Start Time (ES)
  - earliest time an activity can start
  - ES = maximum EF of immediate predecessors

- Earliest finish time (EF)
  - earliest time an activity can finish
  - earliest start time plus activity time

\[ EF = ES + t \]
Earliest Activity Start and Finish Times

- **Start**: 0
- **Design house and obtain financing**: 3
- **Order and receive materials**: 4
- **Lay foundations**:
  - Start: 2
  - Finish: 5
- **Build house**:
  - Start: 4
  - Finish: 8
- **Select pain**:
  - Start: 5
  - Finish: 6
- **Select carpet**: 1
- **Finish work**: 9
Backward Pass

- Determines latest activity times by starting at the end of CPM/PERT network and working forward
- Latest Start Time (LS)
  - Latest time an activity can start without delaying critical path time
    \[ \text{LS} = \text{LF} - t \]
- Latest finish time (LF)
  - Latest time an activity can be completed without delaying critical path time
  - \( \text{LS} = \text{minimum LS of immediate predecessors} \)
Latest Activity Start and Finish Times

- **Start**
- **Design house and obtain financing**
- **Lay foundations**
- **Build house**
- **Order and receive materials**
- **Select paint**
- **Select carpet**
- **Finish work**
**Activity Slack**

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<th>LF</th>
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* Critical Path

**Slack:** amount of time an activity can be delayed without delaying the project

activity slack = \( LS - ES = LF - EF \)

**Critical activities:** have zero slack and lie on a critical path.
Probabilistic Time Estimates

- Beta distribution
  - a probability distribution traditionally used in CPM/PERT

\[
\begin{align*}
\text{Mean (expected time):} & \quad t = \frac{a + 4m + b}{6} \\
\text{Variance:} & \quad \sigma^2 = \left(\frac{b - a}{6}\right)^2 \\
\text{where} & \\
\text{a} & = \text{optimistic estimate} \\
\text{m} & = \text{most likely time estimate} \\
\text{b} & = \text{pessimistic time estimate}
\end{align*}
\]
Examples of Beta Distributions

\[ P(\text{time}) \]

\[ a \quad m \quad t \quad b \]

\[ a \quad m = t \quad b \]

\[ a \quad t \quad m \quad b \]
Project Network with Probabilistic Time Estimates: Example

Start → 1: Equipment installation (6,8,10)

1 → 2: System development (3,6,9)

2 → 3: Position recruiting (1,3,5)

3 → 4: Equipment testing and modification (2,4,12)

4 → 5: System training (3,7,11)

5 → 6: Manual testing (2,3,4)

6 → 7: Job Training (3,4,5)

7 → 8: Orientation (2,2,2)

8 → 9: System testing (2,4,6)

9 → 10: Final debugging (1,4,7)

10 → 11: System changeover (1,10,13)

11 → Finish
# Activity Time Estimates

<table>
<thead>
<tr>
<th>ACTIVITY</th>
<th>TIME ESTIMATES (WKS)</th>
<th>MEAN TIME</th>
<th>VARIANCE</th>
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# Activity Early, Late Times, and Slack

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Earliest, Latest, and Slack

Critical Path

Earliest, Latest, and Slack
Total project variance

$$\sigma^2 = \sigma_2^2 + \sigma_5^2 + \sigma_8^2 + \sigma_{11}^2$$

$$\sigma = 1.00 + 0.11 + 1.78 + 4.00$$

$$= 6.89 \text{ weeks}$$
Probabilistic Network Analysis

Determine probability that project is completed within specified time

\[ Z = \frac{x - \mu}{\sigma} \]

where

- \( \mu = t_p = \text{project mean time} \)
- \( \sigma = \text{project standard deviation} \)
- \( x = \text{proposed project time} \)
- \( Z = \text{number of standard deviations} x \) is from mean
Normal Distribution Of Project Time

\[ \mu = t_p \]

Probability

\[ Z\sigma \]
Probability of Completion Time

What is the probability that the project is completed within 30 weeks?

\[
\begin{align*}
\sigma^2 &= 6.89 \text{ weeks} \\
\sigma &= \sqrt{6.89} \\
\sigma &= 2.62 \text{ weeks}
\end{align*}
\]

\[
Z = \frac{x - \mu}{\sigma} = \frac{30 - 25}{2.62} = 1.91
\]

From Z scores Table, a Z score of 1.91 corresponds to a probability of 0.4719. Thus \( P(30) = 0.4719 + 0.5000 = 0.9719 \)
What is the probability that the project is completed within 22 weeks?

\[ \sigma^2 = 6.89 \text{ weeks} \]
\[ \sigma = \sqrt{6.89} \]
\[ \sigma = 2.62 \text{ weeks} \]

\[ Z = \frac{x - \mu}{\sigma} = \frac{22 - 25}{2.62} = -1.14 \]

From Z scores Table, a Z score of -1.14 corresponds to a probability of 0.3729. Thus \( P(22) = 0.5000 - 0.3729 = 0.1271 \)
Limitations of PERT/CPM

- Assumes clearly defined, independent activities
- Specified precedence relationships
- Activity times (PERT) follow beta distribution
- Subjective time estimates
- Over-emphasis on critical path
  - Monte Carlo Simulations
Project Crashing

- Crashing
  - reducing project time by expending additional resources

- Crash time
  - an amount of time an activity is reduced

- Crash cost
  - cost of reducing activity time

- Goal
  - reduce project duration at minimum cost
Project Crashing: Example
Project Crashing: Example

- Normal activity at $5,000 cost in 12 weeks.
- Normal time = 12 weeks.
- Crash time can reduce the time but at an additional cost.
- Slope = crash cost per week.
- Crashed activity cost is $1,000 more per week than the normal activity.
Normal Activity and Crash Data

<table>
<thead>
<tr>
<th>ACTIVITY</th>
<th>NORMAL TIME (WEEKS)</th>
<th>CRASH TIME (WEEKS)</th>
<th>NORMAL COST</th>
<th>CRASH COST</th>
<th>TOTAL ALLOWABLE CRASH TIME (WEEKS)</th>
<th>CRASH COST PER WEEK</th>
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**Total**  
$75,000  
$110,700
Project Duration: 36 weeks

Project Duration: 31 weeks

Additional Cost: $2000
Time-Cost Relationship

- Crashing costs increase as project duration decreases
- Indirect costs increase as project duration increases
- Reduce project length as long as crashing costs are less than indirect costs
Time-Cost Tradeoff

Minimum cost = optimal project time

Total project cost

Indirect cost

Direct cost

Cost ($)

Crashing

Project duration

Time
References