

# CISC 204 Class 31

## Validity and Consistency in Predicate Logic

Text Correspondence: pp. 131–132

*Main Concepts:*

- *Valid logical system: every theorem is a semantic entailment*
- *Sound logical system: every semantic entailment is a theorem*

The concept of validity, in predicate logic, is an extension of the same concept in propositional logic. In both logics, the sequent  $\vdash \psi$  means that the formula  $\psi$  is a theorem; the entailment  $\models \psi$  means that the formula  $\psi$  is valid. For predicate logic, showing that a theorem is valid is a core concept of semantics.

An important result in predicate logic, which we will not prove because of its difficulty, is that every theorem is satisfiable. This is the *Soundness Theorem*, often written as

$$\text{if } \Gamma \vdash \psi, \text{ then } \Gamma \models \psi$$

The proof of soundness, in propositional logic, required reasoning over every logical operator. The proof of soundness for predicate logic also requires reasoning over quantifiers. We will take this result as known, leaving the proof for more advanced courses.

For our purposes, it is useful to know that if there is a proof of a sequent then the sequent is satisfiable.

Another word for a sound logical system is a *consistent* system.