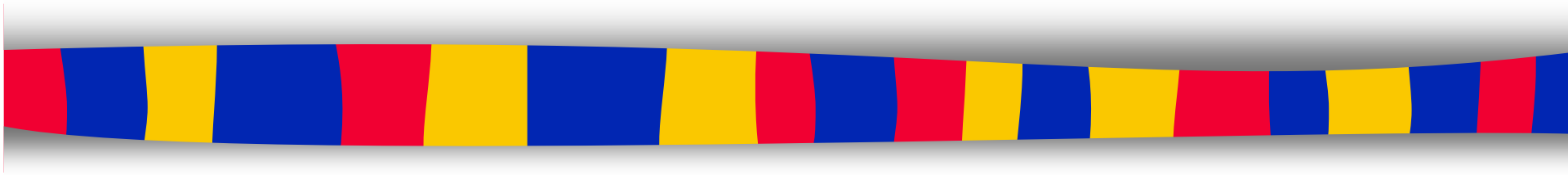
CISC 322

Software Architecture



Lecture 18:

Project Scheduling 2

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Slides adapted from Ahmed E. Hassan

Program Evaluation and Review Technique (PERT)

- **PERT (Program Evaluation and Review Technique)**
  - Based on the idea that estimates are uncertain
  - Uses ranges and probability and provides an expected value to determine duration of the project
  - E.g.
    - The most likely completion time is 4 weeks but
    - It could be anywhere between 3 weeks and 8 weeks

Probabilistic Time Estimates

- Beta distribution
  - a probability distribution traditionally used in CPM/PERT

Mean (expected time): $t = \frac{a + 4m + b}{6}$

Variance: $\sigma^2 = \left(\frac{b - a}{6}\right)^2$

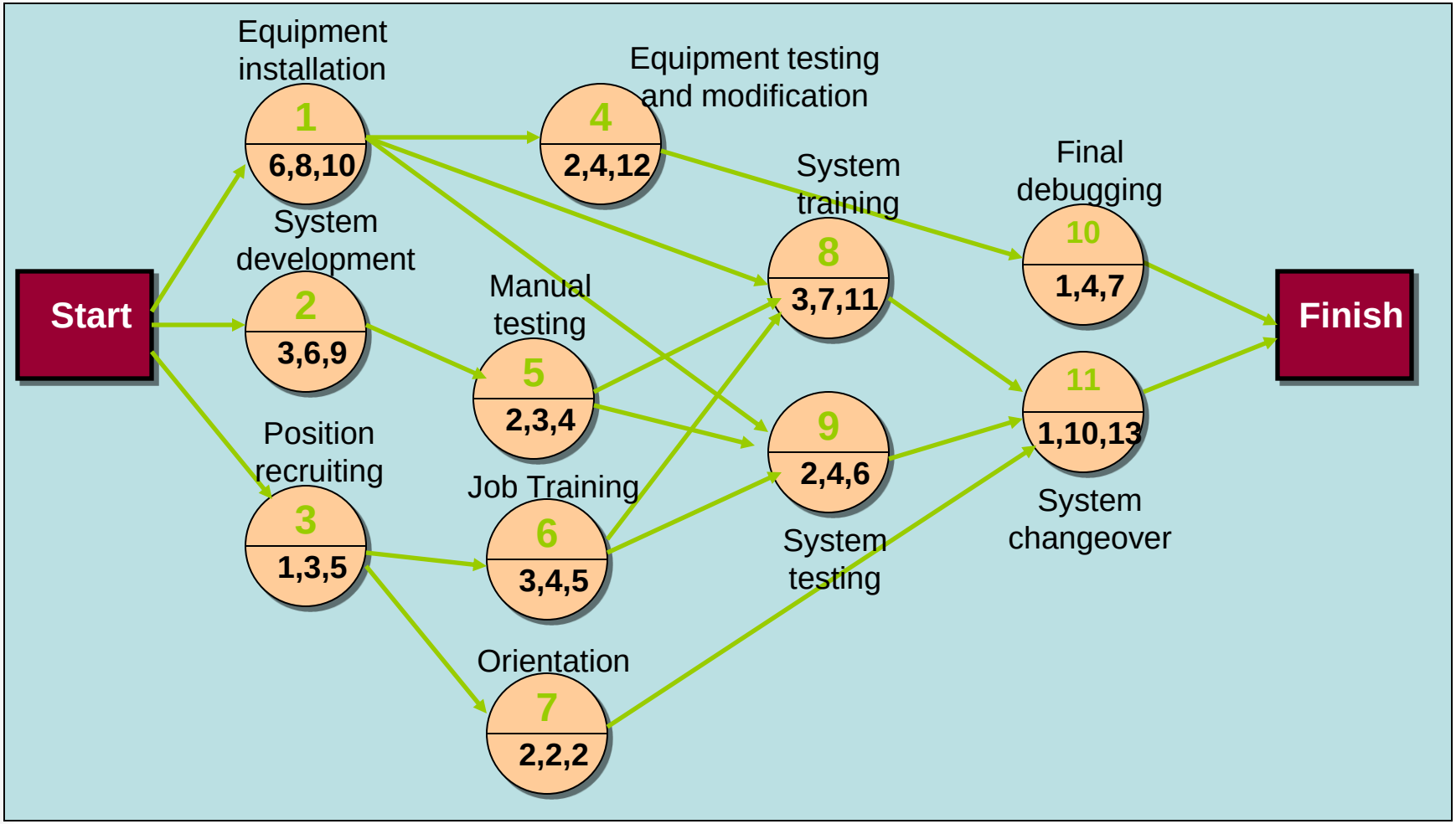
where

a = optimistic estimate

m = most likely time estimate

b = pessimistic time estimate

Project Network with Probabilistic Time Estimates: Example



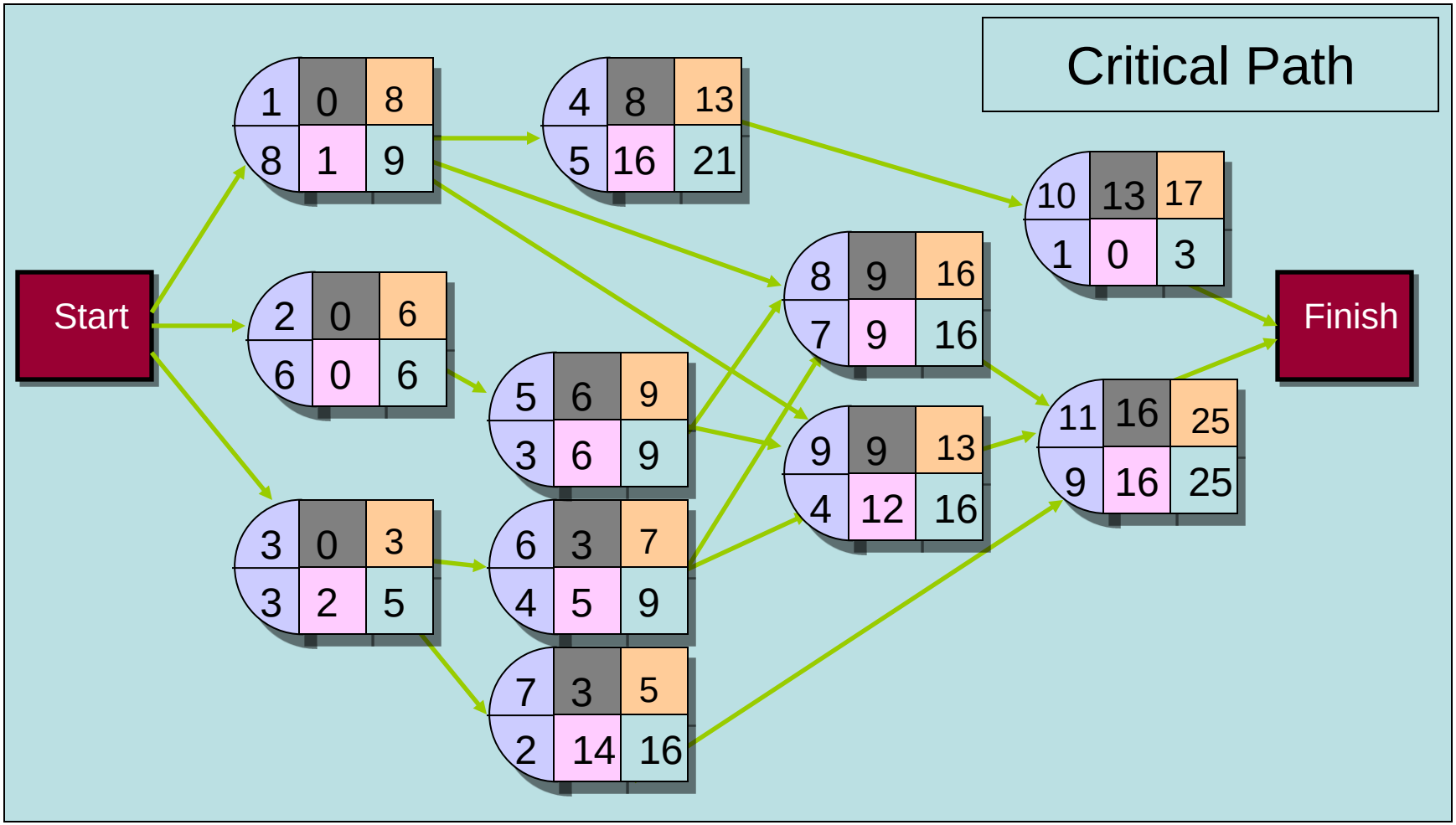
Activity Time Estimates

ACTIVITY	TIME ESTIMATES (WKS)			MEAN TIME	VARIANCE
	a	*m*	*b*	*t*	$б^2$
1	6	8	10	8	0.44
2	3	6	9	6	1.00
3	1	3	5	3	0.44
4	2	4	12	5	2.78
5	2	3	4	3	0.11
6	3	4	5	4	0.11
7	2	2	2	2	0.00
8	3	7	11	7	1.78
9	2	4	6	4	0.44
10	1	4	7	4	1.00
11	1	10	13	9	4.00

Activity Early, Late Times, and Slack

ACTIVITY	t	$б^2$	ES	EF	LS	LF	S
1	8	0.44	0	8	1	9	1
2	6	1.00	0	6	0	6	0
3	3	0.44	0	3	2	5	2
4	5	2.78	8	13	16	21	8
5	3	0.11	6	9	6	9	0
6	4	0.11	3	7	5	9	2
7	2	0.00	3	5	14	16	11
8	7	1.78	9	16	9	16	0
9	4	0.44	9	13	12	16	3
10	4	1.00	13	17	21	25	8
11	9	4.00	16	25	16	25	0

Earliest, Latest, and Slack



Total project variance

$$\sigma^2 = б_2^2 + б_5^2 + б_8^2 + б_{11}^2$$

$$\sigma = 1.00 + 0.11 + 1.78 + 4.00$$

$$= 6.89 \text{ weeks}$$

Probabilistic Network Analysis

Determine probability that project is completed within specified time

$$Z = \frac{x - \mu}{\sigma}$$

where

$\mu = t_p$ = ***project mean time***

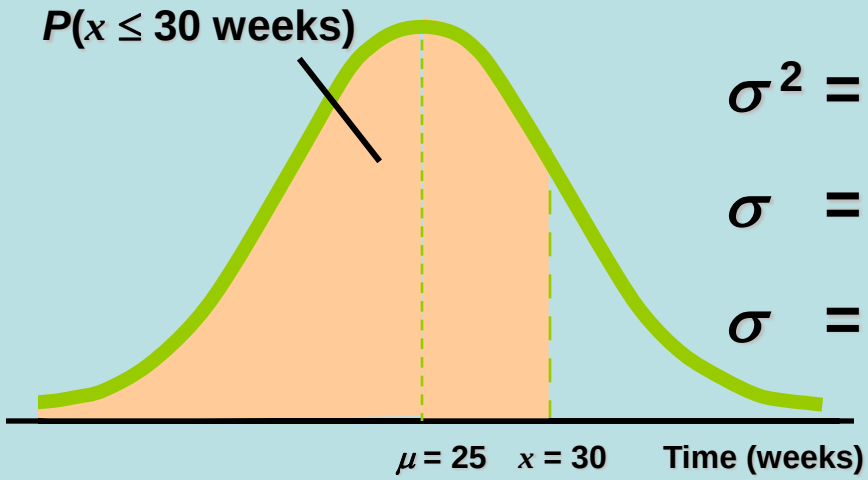
σ = ***project standard deviation***

x = ***proposed project time***

Z = ***number of standard deviations*** x ***is from mean***

Probability of Completion Time

What is the probability that the project is completed within 30 weeks?



$\sigma^2 = 6.89$ weeks

$\sigma = \sqrt{6.89}$

$\sigma = 2.62$ weeks

$$Z = \frac{x - \mu}{\sigma}$$

$$= \frac{30 - 25}{2.62}$$

$$= 1.91$$

From Z scores Table, a Z score of 1.91 corresponds to a probability 0.9719.

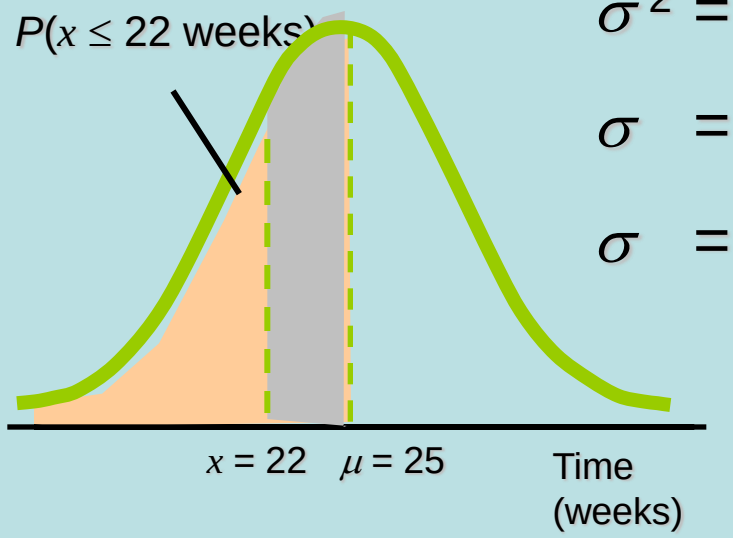
Z values

Cumulative Normal Probability Tables (Z-Values)

Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.50000	0.50399	0.50798	0.51197	0.51595	0.51994	0.52392	0.52790	0.53188	0.53586
0.1	0.53983	0.54380	0.54776	0.55172	0.55567	0.55962	0.56356	0.56749	0.57142	0.57535
0.2	0.57926	0.58317	0.58706	0.59095	0.59483	0.59871	0.60257	0.60642	0.61026	0.61409
0.3	0.61791	0.62172	0.62552	0.62930	0.63307	0.63683	0.64058	0.64431	0.64803	0.65173
0.4	0.65542	0.65910	0.66276	0.66640	0.67003	0.67364	0.67724	0.68082	0.68439	0.68793
0.5	0.69146	0.69497	0.69847	0.70194	0.70540	0.70884	0.71226	0.71566	0.71904	0.72240
0.6	0.72575	0.72907	0.73237	0.73565	0.73891	0.74215	0.74537	0.74857	0.75175	0.75490
0.7	0.75804	0.76115	0.76424	0.76730	0.77035	0.77337	0.77637	0.77935	0.78230	0.78524
0.8	0.78814	0.79103	0.79389	0.79673	0.79955	0.80234	0.80511	0.80785	0.81057	0.81327
0.9	0.81594	0.81859	0.82121	0.82381	0.82639	0.82894	0.83147	0.83398	0.83646	0.83891
1.0	0.84134	0.84375	0.84614	0.84849	0.85083	0.85314	0.85543	0.85769	0.85993	0.86214
1.1	0.86433	0.86650	0.86864	0.87076	0.87286	0.87493	0.87698	0.87900	0.88100	0.88298
1.2	0.88493	0.88686	0.88877	0.89065	0.89251	0.89435	0.89617	0.89796	0.89973	0.90147
1.3	0.90320	0.90490	0.90658	0.90824	0.90988	0.91149	0.91308	0.91466	0.91621	0.91774
1.4	0.91924	0.92073	0.92220	0.92364	0.92507	0.92647	0.92785	0.92922	0.93056	0.93189
1.5	0.93319	0.93448	0.93574	0.93699	0.93822	0.93943	0.94062	0.94179	0.94295	0.94408
1.6	0.94520	0.94630	0.94738	0.94845	0.94950	0.95053	0.95154	0.95254	0.95352	0.95449
1.7	0.95543	0.95637	0.95728	0.95818	0.95907	0.95994	0.96080	0.96164	0.96246	0.96327
1.8	0.96407	0.96485	0.96562	0.96638	0.96712	0.96784	0.96856	0.96926	0.96995	0.97062
1.9	0.97128	0.97193	0.97257	0.97320	0.97381	0.97441	0.97500	0.97558	0.97615	0.97670
2.0	0.97725	0.97778	0.97831	0.97882	0.97932	0.97982	0.98030	0.98077	0.98124	0.98169
2.1	0.98214	0.98257	0.98300	0.98341	0.98382	0.98422	0.98461	0.98500	0.98537	0.98574
2.2	0.98610	0.98645	0.98679	0.98713	0.98745	0.98778	0.98809	0.98840	0.98870	0.98899
2.3	0.98928	0.98956	0.98983	0.99010	0.99036	0.99061	0.99086	0.99111	0.99134	0.99158
2.4	0.99180	0.99202	0.99224	0.99245	0.99266	0.99286	0.99305	0.99324	0.99343	0.99361
2.5	0.99379	0.99396	0.99413	0.99430	0.99446	0.99461	0.99477	0.99492	0.99506	0.99520
2.6	0.99534	0.99547	0.99560	0.99573	0.99585	0.99598	0.99609	0.99621	0.99632	0.99643
2.7	0.99653	0.99664	0.99674	0.99683	0.99693	0.99702	0.99711	0.99720	0.99728	0.99736
2.8	0.99744	0.99752	0.99760	0.99767	0.99774	0.99781	0.99788	0.99795	0.99801	0.99807
2.9	0.99813	0.99819	0.99825	0.99831	0.99836	0.99841	0.99846	0.99851	0.99856	0.99861

Probability of Completion Time

What is the probability that the project is completed within 22 weeks?



$\sigma^2 = 6.89$ weeks

$\sigma = \sqrt{6.89}$

$\sigma = 2.62$ weeks

$$Z = \frac{x - \mu}{\sigma}$$

$$= \frac{22 - 25}{2.62}$$

$$= -1.14$$

From Z scores Table, a Z score of -1.14 corresponds to a probability of 0.1271

Z values

Cumulative Normal Probability Tables (Z-Values)

Z	0.09	0.08	0.07	0.06	0.05	0.04	0.03	0.02	0.01	0.00
-4.0	0.00002	0.00002	0.00002	0.00002	0.00003	0.00003	0.00003	0.00003	0.00003	0.00003
-3.9	0.00003	0.00003	0.00004	0.00004	0.00004	0.00004	0.00004	0.00004	0.00005	0.00005
-3.8	0.00005	0.00005	0.00005	0.00006	0.00006	0.00006	0.00006	0.00007	0.00007	0.00007
-3.7	0.00008	0.00008	0.00008	0.00008	0.00009	0.00009	0.00010	0.00010	0.00010	0.00011
-3.6	0.00011	0.00012	0.00012	0.00013	0.00013	0.00014	0.00014	0.00015	0.00015	0.00016
-3.5	0.00017	0.00017	0.00018	0.00019	0.00019	0.00020	0.00021	0.00022	0.00022	0.00023
-3.4	0.00024	0.00025	0.00026	0.00027	0.00028	0.00029	0.00030	0.00031	0.00032	0.00034
-3.3	0.00035	0.00036	0.00038	0.00039	0.00040	0.00042	0.00043	0.00045	0.00047	0.00048
-3.2	0.00050	0.00052	0.00054	0.00056	0.00058	0.00060	0.00062	0.00064	0.00066	0.00069
-3.1	0.00071	0.00074	0.00076	0.00079	0.00082	0.00084	0.00087	0.00090	0.00094	0.00097
-3.0	0.00100	0.00104	0.00107	0.00111	0.00114	0.00118	0.00122	0.00126	0.00131	0.00135
-2.9	0.00139	0.00144	0.00149	0.00154	0.00159	0.00164	0.00169	0.00175	0.00181	0.00187
-2.8	0.00193	0.00199	0.00205	0.00212	0.00219	0.00226	0.00233	0.00240	0.00248	0.00256
-2.7	0.00264	0.00272	0.00280	0.00289	0.00298	0.00307	0.00317	0.00326	0.00336	0.00347
-2.6	0.00357	0.00368	0.00379	0.00391	0.00402	0.00415	0.00427	0.00440	0.00453	0.00466
-2.5	0.00480	0.00494	0.00508	0.00523	0.00539	0.00554	0.00570	0.00587	0.00604	0.00621
-2.4	0.00639	0.00657	0.00676	0.00695	0.00714	0.00734	0.00755	0.00776	0.00798	0.00820
-2.3	0.00842	0.00866	0.00889	0.00914	0.00939	0.00964	0.00990	0.01017	0.01044	0.01072
-2.2	0.01101	0.01130	0.01160	0.01191	0.01222	0.01255	0.01287	0.01321	0.01355	0.01390
-2.1	0.01426	0.01463	0.01500	0.01539	0.01578	0.01618	0.01659	0.01700	0.01743	0.01786
-2.0	0.01831	0.01876	0.01923	0.01970	0.02018	0.02068	0.02118	0.02169	0.02222	0.02275
-1.9	0.02330	0.02385	0.02442	0.02500	0.02559	0.02619	0.02680	0.02743	0.02807	0.02872
-1.8	0.02938	0.03005	0.03074	0.03144	0.03216	0.03288	0.03362	0.03438	0.03515	0.03593
-1.7	0.03673	0.03754	0.03836	0.03920	0.04006	0.04093	0.04182	0.04272	0.04363	0.04457
-1.6	0.04551	0.04648	0.04746	0.04846	0.04947	0.05050	0.05155	0.05262	0.05370	0.05480
-1.5	0.05592	0.05705	0.05821	0.05938	0.06057	0.06178	0.06301	0.06426	0.06552	0.06681
-1.4	0.06811	0.06944	0.07078	0.07215	0.07353	0.07493	0.07636	0.07780	0.07927	0.08076
-1.3	0.08226	0.08379	0.08534	0.08692	0.08851	0.09012	0.09176	0.09342	0.09510	0.09680
-1.2	0.09853	0.10027	0.10204	0.10383	0.10565	0.10749	0.10935	0.11123	0.11314	0.11507
-1.1	0.11702	0.11900	0.12100	0.12302	0.12507	0.12714	0.12924	0.13136	0.13350	0.13567
-1.0	0.13786	0.14007	0.14231	0.14457	0.14686	0.14917	0.15151	0.15386	0.15625	0.15866
-0.9	0.16109	0.16354	0.16602	0.16853	0.17106	0.17361	0.17619	0.17879	0.18141	0.18406

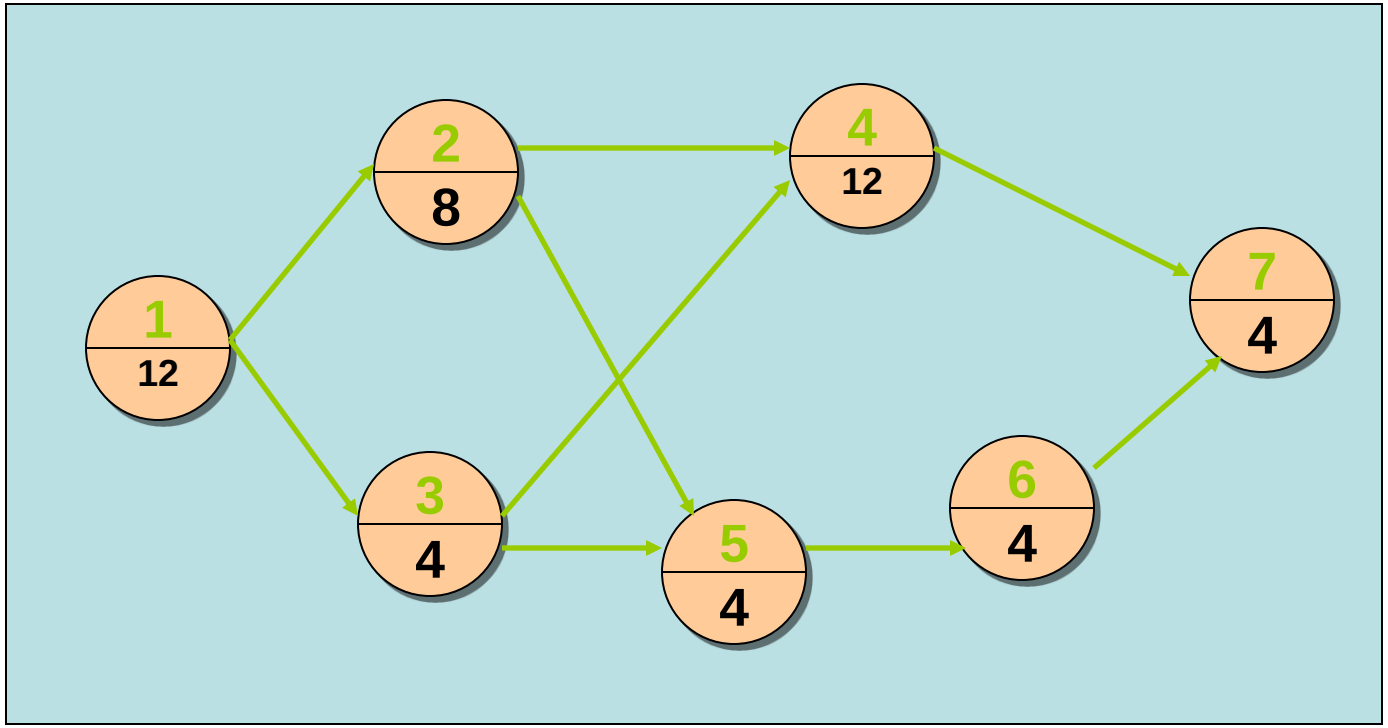
Limitations of PERT/CPM

- Assumes clearly defined, independent activities
- Activity times (PERT) follow beta distribution
- Subjective time estimates
- Over-emphasis on critical path

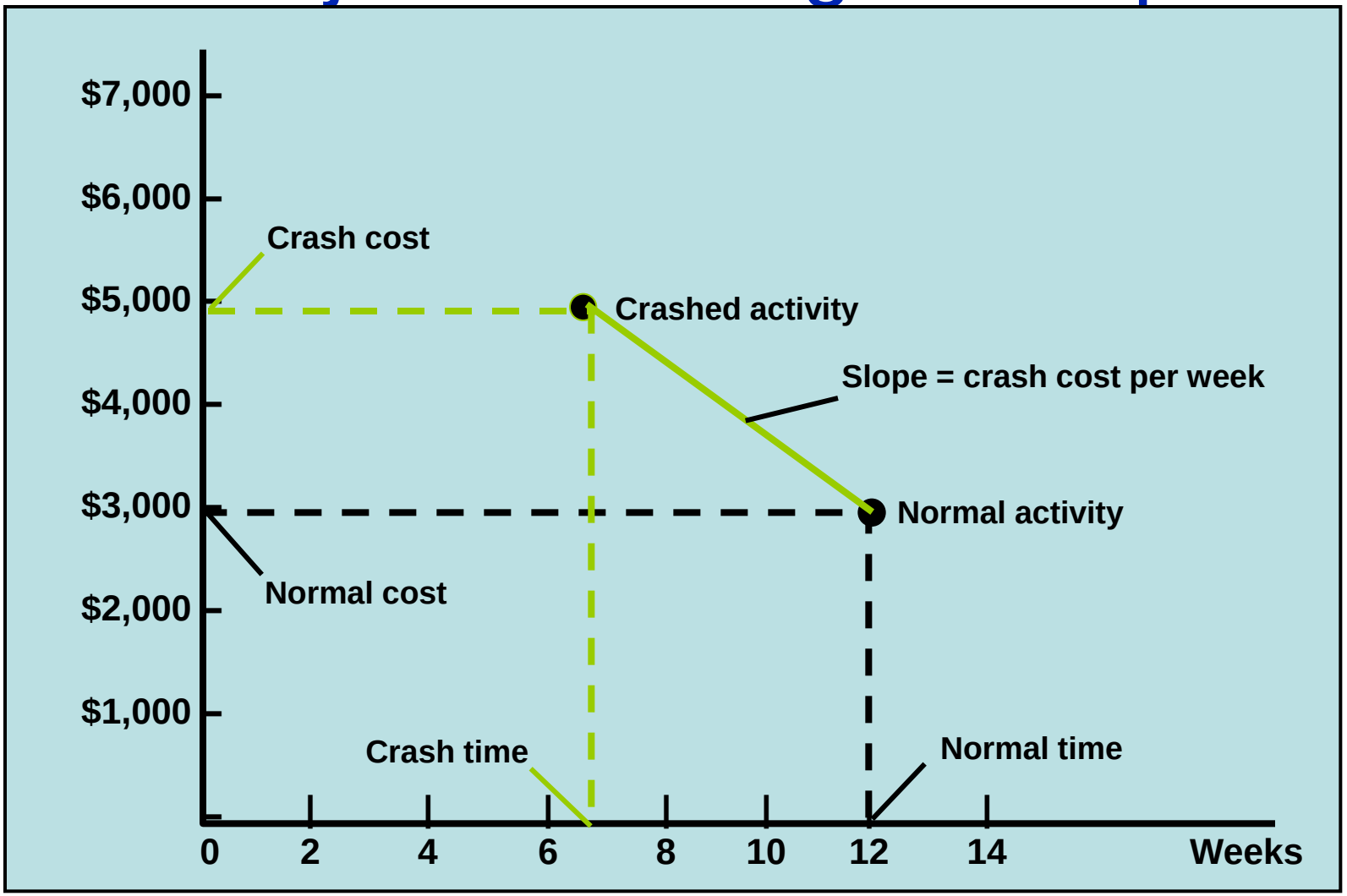
Project Crashing

- Crashing
  - reducing project time by expending additional resources
- Crash time
  - an amount of time an activity is reduced
- Crash cost
  - cost of reducing activity time
- Goal
  - reduce project duration at minimum cost

Project Crashing: Example

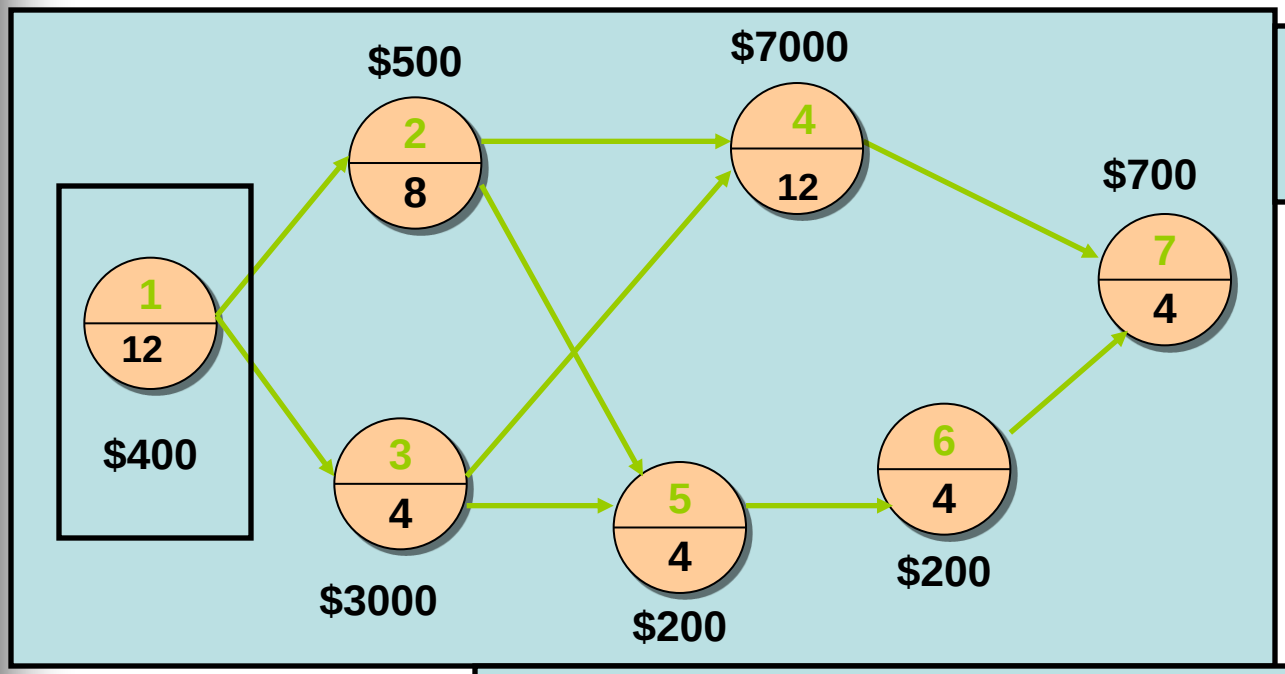


Project Crashing: Example



Normal Activity and Crash Data

ACTIVITY	NORMAL TIME (WEEKS)	CRASH TIME (WEEKS)	NORMAL COST	CRASH COST	TOTAL ALLOWABLE CRASH TIME (WEEKS)	CRASH COST PER WEEK
1	12	7	$3,000	$5,000	5	$400
2	8	5	2,000	3,500	3	500
3	4	3	4,000	7,000	1	3,000
4	12	9	50,000	71,000	3	7,000
5	4	1	500	1,100	3	200
6	4	1	500	1,100	3	200
7	4	3	15,000	22,000	1	7,000
			$75,000	$110,700		

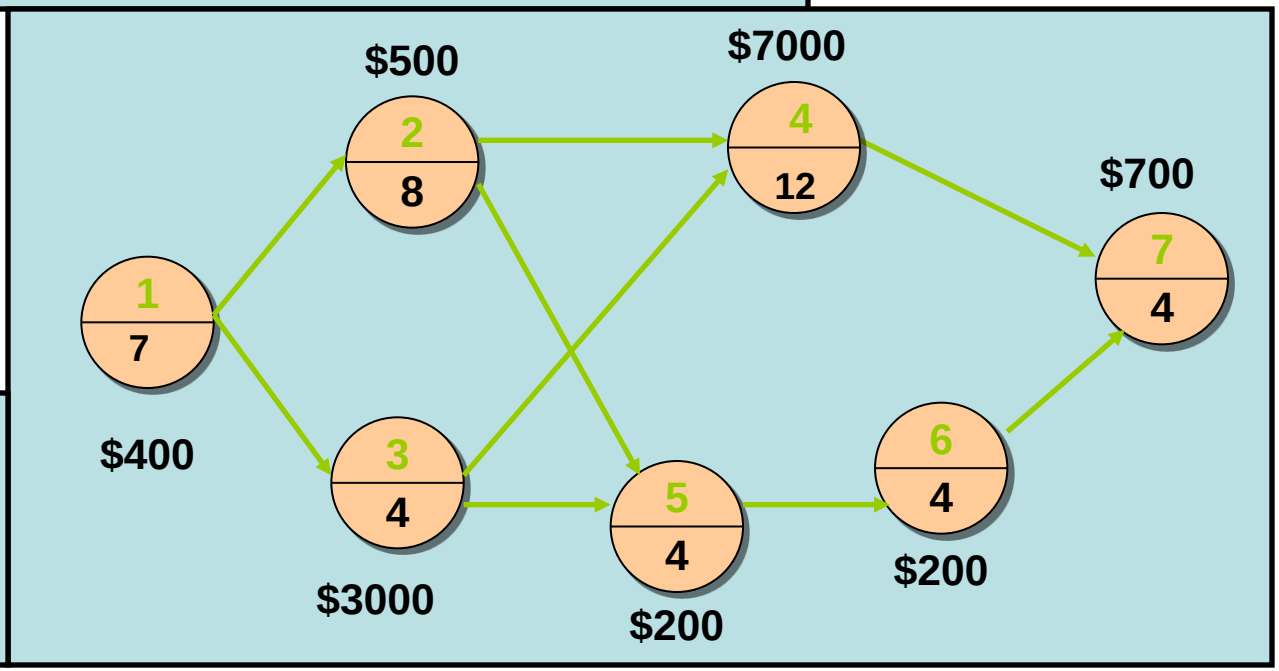


Project Duration: 36 weeks

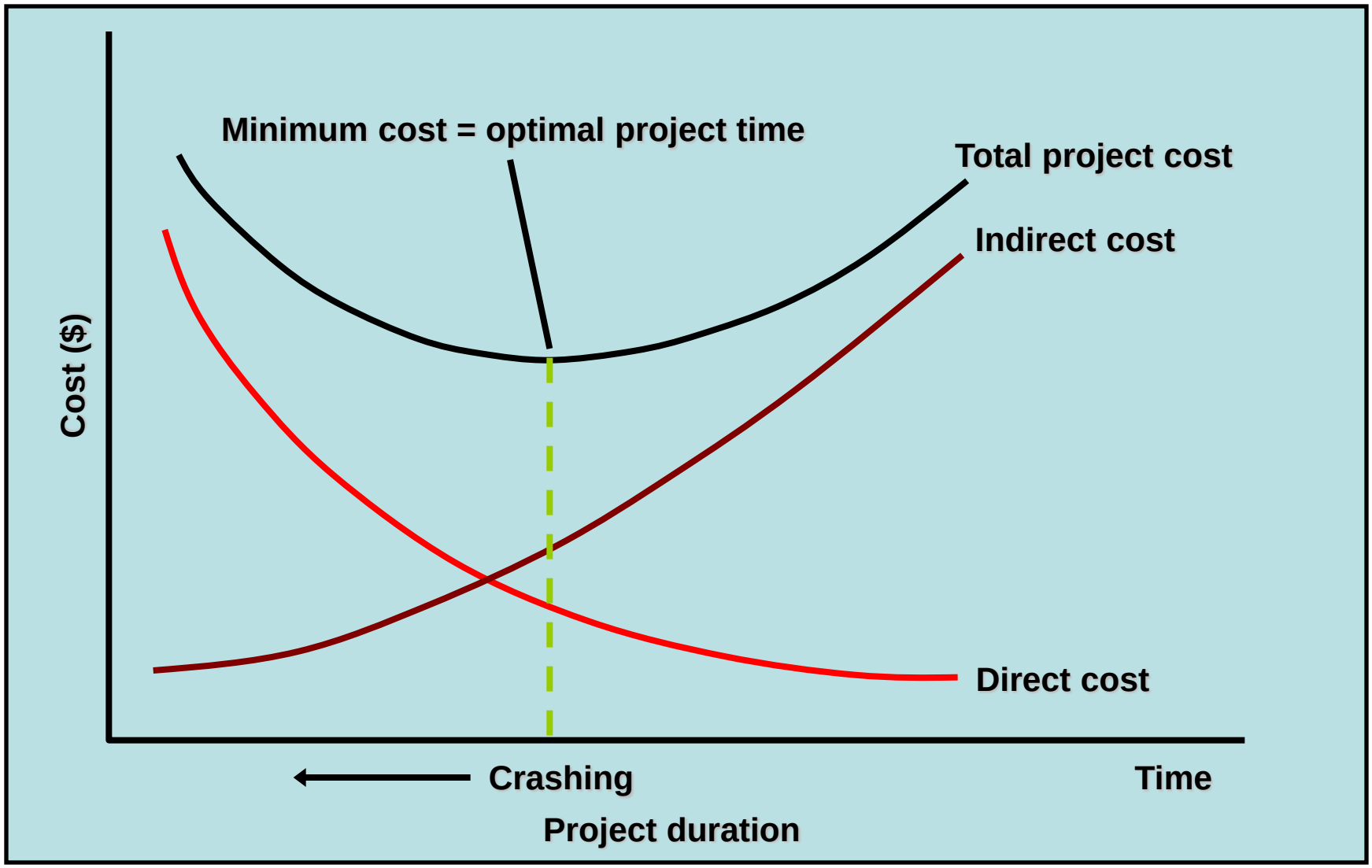
FROM …

TO…

Project Duration: 31 weeks
Additional Cost: $2000



Time-Cost Tradeoff



Time-Cost Relationship

- Crashing costs increase as project duration decreases
- Indirect costs increase as project duration increases
- Reduce project length as long as crashing costs are less than indirect costs

References

- Hughes, B., and Cotterell, M. (1999) *Software Project Management*, 2nd edition, McGraw-Hill. (slides)
- Pfleeger, S.L. (1998) *Software Engineering: Theory and Practice*, Prentice Hall.
- Roberta Russell & Bernard W. Taylor, III (2006) Operations Management - 5th Edition, John Wiley & Sons (slides)
- http://miha.ef.uni- lj.si/_dokumenti3plus2/195166/norm-tables.pdf