lec11: Equivalence, Identity, and the Proof Game

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1 Equivalence and identity

[handwritten notes]

2 The Proof Game

3 The Proof Game: Rulebook

This is a cooperative game in which we attempt to reach a goal given by the referee (me).

Each turn consists of the referee calling on a player to suggest a move. (This is less unpleasant than it may sound.)

You can suggest a move *only if* you hold a "card" (actually just a piece of paper) with that move printed on it. Because the number of cards is very limited, you will have to make some suggestions that are not useful, or even impossible. I will explain why the move would not make progress towards the goal, and I will call on another player.

Some (not all) moves demonstrate new facts, which I will write on the board.

Not all moves will be available today.

3.1 Move 0: notice that the goal has been reached

This is the move that "wins" the game. For example, if our goal is to prove $(\texttt{+}\ 0\ 0) \Downarrow 0$, and $(\texttt{+}\ 0\ 0) \Downarrow 0$ is on the board, making this move wins. The game is not over until this move is made.

3.2 Move 1: apply a rule "forward", from premises to conclusion

If all the premises of a rule are on the board, this move demonstrates the conclusion. It also builds a derivation, but I will not necessarily write the derivation, just the conclusion.

3.3 Move 2: apply an equivalence ($=_{eqv}$)

If we have shown that two things defined prior to this course are equivalent (equal), we can replace them in a known fact to get a "new" fact. For example, if we have a fact involving $2 + \pi - \pi$ we can replace it with 2 to get a new fact, because we agree that $2 + \pi - \pi =_{eqv} 2$.

3.4 Move 3: apply an identity ($=_{id}$)

If we have shown that two things defined using grammars are identical, we can replace them in a known fact to get a "new" fact.

3.5 Move 4: use inversion

If we have a derivation of a judgment, and want to use "inverted" reasoning to obtain derivations of its premises and equations ($=_{id}$), we use inversion. For example, the only way to derive $(\texttt{Abs}\ e_1) \Downarrow 5$ is by rule eval-abs, which then tells us that $e_1 \Downarrow n$ where $|n| = 5$.

3.6 Move 5: use case analysis

For example, if we are given expressions e and v and our goal is to show $v = 0$, we can case-analyze (or "case-split") e, breaking the proof into three pieces: one where $e = n$, one where $e = (\texttt{+}\ e_1\ e_2)$, and one where $e = (\texttt{Abs}\ e_1)$.

3.7 Move 6: observe a $\prec$ (subexpression or subderivation)

3.8 Move 7: apply the IH