CISC-102

HOMEWORK 2

Please work on these problems and have your solutions ready by January 21. Assignments will not be collected for grading.

Readings

Read sections 1.5, 1.6, 1.7, 1.8 of *Schaum’s Outline of Discrete Mathematics.*
Read sections 1.1, 1.2 and 1.3 again (if you did not understand things last week) of *Discrete Mathematics Elementary and Beyond.*

Problems

(1) Illustrate DeMorgan’s Law \((A \cap B)^c = A^c \cup B^c\) using Venn diagrams.

(2) Let \(A_i = \{1, 2, 3, \ldots, i\}\) for all \(i \in \mathbb{N}\). For example \(A_4 = \{1, 2, 3, 4\}\).
   (a) What are the elements of the set \(\bigcup_{i=1}^{n} A_i\)?
   (b) What are the elements of the set \(\bigcap_{i=1}^{n} A_i\)?

(3) Observe that \(A \subseteq B\) has the same meaning as \(A \cap B = A\). Draw a Venn diagram to illustrate this fact.

(4) Use a Venn diagram to show that if \(A \subseteq B\) and \(B \subseteq C\), then \(A \subseteq C\).

(5) Use the Principle of Exclusion and Inclusion to show that \(|A \cup B| + |A \cap B| = |A| + |B|\).
   (It may help your understanding if you first explore an example such as \(A = \{1, 2, 3\}\) and \(B = \{3, 4\}\).)

(6) What are the cardinalities of the following sets?
   (a) \(A = \{\text{winter, spring, summer, fall}\}\).
   (b) \(B = \{x : x \in \mathbb{Z}, 0 < x < 7\}\).
   (c) \(\mathcal{P}(B)\), that is, the power set of B.
   (d) \(C = \{x : x \in \mathbb{N}, x \text{ is even}\}\).

(7) Suppose that we have a sample of 100 students at Queen’s who take at least one of the following language courses, French-101, Spanish-101, German-101. Also suppose that 65 take French-101, 45 take German-101, 42 take Spanish-101, 20 take French-101 and German-101, 25 take French-101 and Spanish-101, 15 take German-101 and Spanish-101.
(a) How many students take all three language courses? (HINT: Use the Principle of Inclusion and Exclusion to write an expression representing these students and the classes they take.)

(b) Draw a Venn diagram representing these 100 students and fill in the regions with the correct number.

(c) How many students take exactly 1 of these courses?

(d) How many students take exactly 2 of these courses?

(8) At an art class with 30 students, there are 14 women, and 16 men. Twenty-two of the students are right-handed. What is the minimum and maximum number of women that are right-handed?

(9) Recall that the union operation is associative, that is $A \cup (B \cup C) = (A \cup B) \cup C$. Show that the relative complement set operation is not associative, that is, $A \setminus (B \setminus C) = (A \setminus B) \setminus C$, is incorrect for some sets $A$, $B$, $C$. (Note if relative complement is associative then the equation must be true for all sets $A$, $B$, $C$.)

(10) Consider a set $S$ of $n$ elements, such that $\{a, b\} \subseteq S$.

(a) What is the cardinality of the power set of $S \setminus \{a\}$?

(b) What is the cardinality of the power set of $S \setminus \{a, b\}$?

(c) How many subsets of $S$ are there that contain the element $a$?

(d) How many subsets of $S$ are there that contain the element $a$ and exclude the element $b$?