Please write your answer to each question only in the box marked Answer.

No questions will be answered by the instructor during the exam.

This is a closed-book exam. No computers, calculators, cell phones or scrap paper are allowed.

Use the back of pages for scratch work.

If you are unsure of what is wanted for a particular question, make a reasonable assumption and write this at the beginning of your answer.

PLEASE NOTE: Proctors are unable to respond to queries about the interpretation of exam questions. Do your best to answer exam questions as written.

NAME: ________________________________

STUDENT NUMBER: __________________

FOR INSTRUCTOR’S USE ONLY

Question 1: ______ / 6

Question 2: ______ / 5

Question 3: ______ / 6

Question 4: ______ / 5

TOTAL: ______ / 22
Question 1: [6 marks]

List all the elements of the following sets:

(i) \( A = \{ x : x \in \mathbb{N}, 1 \leq x \leq 7 \} \)
Answer:
\[ A = \{ 1, 2, 3, 4, 5, 6, 7 \} \]

(ii) \( B = \{ x : x \in \mathbb{Z}, x^2 - 1 = 0 \} \)
Answer:
\[ B = \{ -1, 1 \} \]

(iii) \( C = \{ x : x \in \mathbb{Q}, x^2 - 4 = 0 \} \)
Answer:
\[ C = \{ -2, 2 \} \]

(iv) \( D = \{ x : x \in \mathbb{R}, x^2 + 4 = 0 \} \)
Answer:
\[ D = \emptyset \]
Question 2: [5 marks]

I. Consider the sets $X = \{1, 3, 5, 7\}, Y = \{3, 4, 5, 6\}, Z = \{3, 7\}$.

(i) Suppose $A \subseteq B$. Which of the sets above can be $A$ and which can be $B$? List all possibilities if more than one.
Answer:

\[ X \subseteq X, Y \subseteq Y, Z \subseteq Z, Z \subseteq X \]

(ii) Suppose $C \supset D$. Which of the sets above can be $C$ and which can be $D$? List all possibilities if more than one.
Answer:

\[ X \supset Z \]

(iii) Let $E$ be a set that is a subset of $X \cap Y \cap Z$ with the most elements. What are the elements of $E$?
Answer:

\[ E = \{3\} \]

(iv) Let $F$ be a set that is a subset of $X \cup Y \cup Z$ with the fewest elements. What are the elements of $F$?
Answer:

\[ F = \emptyset \]

II. Ten households responded to a survey on the pets that they had. Six responded that they had dogs, four said that they had cats, and two replied that they had neither cats nor dogs. Using these responses, find the number of households with both cats and dogs.

Answer:
Let $x$ be the number of households with both cats and dogs.

\[ 6 + 4 - x = 10 - 2 \]

Therefore, $x = 2$. 
Question 3: [6 marks]

Recall that the *relative complement* of a set $B$ with respect to a set $A$, denoted by $A \setminus B$, and the *symmetric difference* of sets $A$ and $B$, denoted by $A \oplus B$, are defined as:

$$A \setminus B = \{ x : x \in A, x \notin B \}, A \oplus B = (A \cup B) \setminus (A \cap B).$$

Let $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, $A = \{1, 3, 4\}$, $B = \{2, 3, 4, 5\}$, and $C = \{3, 4\}$.

(i) What is $A \setminus B$?
Answer:

$$A \setminus B = \{1\}$$

(ii) What is $A \oplus B$?
Answer:

$$A \oplus B = \{1, 2, 5\}$$

(iii) What is $(B \setminus C) \oplus A$?
Answer:

$$B \setminus C = \{2, 5\}$$

$$(B \setminus C) \oplus A = \{1, 2, 3, 4, 5\}$$
Question 4: [5 marks]

Let $A = \{\{a\}, \{b, c, d, e\}, \{c, d\}\}$. For each of the following statements indicate whether it is true or false, by writing either T or F beside each statement.

<table>
<thead>
<tr>
<th>Answer:</th>
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| (i) $a \in A$ **F**  
  (Note: $\{a\} \in A$) |
| (ii) $\{\{a\}, \{c, d\}\} \subseteq A$ **T** |
| (iii) $\emptyset \in A$ **F** |
| (iv) $\emptyset \subseteq A$ **T** |
| (v) $\emptyset \subset A$ **T** |
| (vi) $\{a, b, c, d, e\} \subset A$ **F** |