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ON THE IMPORTANCE OF BEING QUANTUM*

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Abstract

Game playing is commonly cited in debates concerning human versus machine intelligence, and Chess is often at the center of such debates. However, the role of Chess in delineating the difference between natural and artificial intelligence has been significantly diminished since a World Chess Champion lost in a tournament against a computer. Computer brute force is regularly blamed for the human defeat. This paper proposes a Quantum Chess Board in an attempt to bring back some equilibrium, putting humans and computers on an ostensibly equal footing when faced with the uncertainties of quantum physics.

Keywords: Chess, game-playing programs, unconventional computation, quantum computing.

1 INTRODUCTION

"It was at this time that the game of chess was invented, which eclipsed backgammon by demonstrating how intelligence brings success and ignorance failure."
Mas'ūdī, *The Meadows of Gold*

This paper represents a modest contribution to the debate regarding human versus machine intelligence. Since the dawn of computing, beginning with the writings of Alan Turing (for an excellent reference, see Turing's collected works in [7]), game playing has been considered as a valid test of whether a computer program has demonstrated a certain form of intelligence. Early workers in the field of *artificial intelligence* focused their efforts on programs that played games against humans and other programs [21]. The game of Chess, with its illustrious history, the legends surrounding it, and its special mystique, was considered the game of strategy par excellence and the true test of a machine's ability to compete

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against a human in an activity that required reason [10, 28]. The matter appeared to have been settled on May 11, 1997, when a special-purpose Chess-playing computer (developed by IBM and called *Deep Blue*) defeated a Chess Grandmaster and World Chess Champion (Garry Kasparov) [15].

However, doubts still linger. There are those who claim that the aforementioned match was unfair and included various irregularities on the part of the organizers (for details and a response, see [9]). A more substantial criticism of the outcome, however, is the claim that the machine that won the match resorted to brute force–massive parallel information processing, in contrast with the refined, logical way that (presumably) characterizes human play–and consequently its performance does not in any way qualify as an intelligence comparable to a human's [26].

The purpose of this paper is to propose a variant of the game of Chess that restores the balance and evens the chances between humans and computers. Our motivation for seeking such a variant stems from the fact that Chess is a non-chance game of complete information. At each step of the game, the player whose turn it is to move, has total control and can choose that move from among several with certainty. This is helped by the fact that each of the two players, at any moment during the game, has complete knowledge of

1. All the past moves played by both players from the beginning of the game up to that point,
2. The current configuration of the pieces on the board,
3. All possible future moves by both players, in theory, until the end of the game.

This in essence is a very simple game for a computer, and a trivial one for a powerful computer: Assuming the game is modeled as a tree whose nodes are board positions, the root being the current board position, and whose edges represent moves, the computer can explore all moves and the replies to these moves, and the replies to the replies, and so on, generating in the process a tree, of all theoretically possible games. Some refined algorithms allow this giant tree to be 'pruned', and the whole tree is never really generated, but the idea that a (nearly) exhaustive search is advantageous explains why computers are defeating the vast majority of, if not all, humans in the game of Chess. Therefore, games of *incomplete* information present a greater challenge. The new game proposed here meets this requirement. It is called *Quantum Chess*, as it is played on a board that obeys the laws of quantum physics. Superposition, uncertainty, interference, and entanglement, all contribute to make Quantum Chess the great equalizer between human and machine.

It is important at the outset to clarify what this paper is *not* about:

1. We are not concerned with *quantum game theory* which (as described, for example, in [5, 11, 17, 20]) is a field of study that applies the properties of quantum physics in an abstract manner with the purpose of making decisions in adversarial situations.
2. Furthermore, bringing quantum physics into Chess should be understood as being significantly different from merely introducing to the game an element of chance, the

latter manifesting itself, for example, in games involving dice or playing cards, where all the possible outcomes and their odds are known in advance.

3. Similarly, while this paper deals with questions of quantum computation and human and machine intelligence, it does not address the question of whether the human brain is a quantum computer, as claimed in [18, 19].

4. As well, there is no connection between this work and variants on the game of Chess, that do not use any properties of quantum physics in theory or in practice, but are nevertheless called "Quantum Chess", because they are harder than traditional Chess by a "quantum leap" [22, 23], or because new rules are used whose *names* are inspired by quantum physics [24], or curiously for no apparent reason at all [25].

5. Finally, the phrase "quantum chess" appears in contexts without any relationship with the game of Chess itself. Thus, for example, it is used in [8] to describe a lattice (resembling a chess board and representing space-time) on which quantum chromodynamics computations are performed that involve simulating the motion of quarks and gluons. Similarly, the name "Quantum Chess Board" in [27] refers to a pattern of light and dark squares arising in a particular quantum optics application and suggestive of a Chess board.

We assume in what follows that the reader has some familiarity with the game of Chess as well as the rudiments of quantum computation and quantum information [14, 16]. The remainder of the paper is organized as follows. Section 2 offers some preliminary ideas that serve as background and motivation for the subsequent material. Quantum Chess is introduced in Section 3. In Section 4 we explore the interesting question of whether a quantum computer would have an advantage in playing Quantum Chess against a human opponent. The paper concludes in Section 5 with a few remarks on the role of quantum physics in the game-playing-as-intelligence debate.

2 PRELIMINARIES

"The king worked out mathematical models for chess
and composed a book on this subject called
Taraq Jankā, *still popular among indians."*
Mas'ūdī, *The Meadows of Gold*

This section describes a number of initial attempts at defining a quantum variant of Chess. We begin by introducing a helpful notation.

2.1 Notation

Conventional Chess is played on a square board divided into 64 small squares of equal size (arranged in eight rows and eight columns in a regular pattern of alternating black and white squares). There are two sets of Chess pieces, one black and one white, initially placed on the top two rows and bottom two rows of the board, respectively, one piece per square. One player plays with the white pieces, the other with the black [28].

For ease of exposition, we now introduce an unconventional notation for Chess pieces. Also for simplicity, we use generic symbols for the two sets of pieces, without distinction between black and white. The notation can be easily extended by adding a 'B' or a 'W' prefix, as needed. Each player receives 16 pieces:

1. Eight Pawns, whose motion is forward or diagonally (when capturing) by one square, denoted here $P_1, P_2, \ldots, P_8$.

2. Two Rooks whose motion is horizontal or vertical by several squares, the Left Rook and the Right Rook, denoted here R_l and R_r, respectively.

3. Two Knights whose motion is in an **L** pattern of three squares (and its rotations), the Left Knight and the Right Knight, denoted here N_l and N_r, respectively.

4. Two Bishops whose motion is diagonal by several squares, one on white squares, the other on black squares, the Left Bishop and the Right Bishop, denoted B_l and B_r, respectively.

5. The Queen, whose motion combines the motions of the two Rooks and two Bishops, denoted here Q.

6. The King, whose motion is almost identical to that of a Pawn, except that it can retreat, denoted here K.

All pieces other than the Pawn capture an opponent's piece if it is on the square on which they land after a move. There are also small variants and exceptions to the above rules, that are of no consequence here. The purpose of the game is to capture the opponent's King. The initial arrangement of the pieces, on the top two rows (the black pieces, for example) is:

$$R_l N_l B_l Q K B_r N_r R_r$$

$$P_1 P_2 P_3 P_4 P_5 P_6 P_7 P_8$$

while the initial arrangement of the pieces on the bottom two rows (the white pieces, for example) is:

$$P_1 P_2 P_3 P_4 P_5 P_6 P_7 P_8$$

$$R_l N_l B_l Q K B_r N_r R_r.$$

We note here in passing that once the pieces are placed on a Chess board in their initial positions, the color of the squares (black and white) on the board are entirely without significance; each piece has its rules of motion and capture, all squares could be the same color and the game would still be played perfectly.

2.2 Let's Bring Quantum Physics To Chess

In introducing quantum physics into Chess, we seek our inspiration from the Queen piece in conventional Chess, which combines the properties of the Rook and Bishop pieces, so that the player can decide to move the Queen vertically, horizontally, on a white diagonal, or on a black one.

2.2.1 Variant 1

Suppose each piece is in a quantum superposition with another piece. Thus if R_l and B_r, are in a superposition, we denote such a superposition with $[R_l/B_r]$. Thus all possible superpositions of P_1 are:

$$[P_1/P_2], [P_1/P_3], \ldots, [P_1/P_8], [P_1/R_l], [P_1/N_l], \ldots, [P_1/R_r],$$

all possible superpositions of P_2 are:

$$[P_2/P_1], [P_2/P_3], \ldots, [P_2/P_8], [P_2/R_l], [P_2/N_l], \ldots, [P_2/R_r],$$

and so on, until all possible superpositions of R_r:

$$[R_r/P_1], [R_r/P_2], \ldots, [R_r/P_8], [R_r/R_l], [R_r/N_l], \ldots, [R_r/N_r].$$

For each game, each piece is in one of these superpositions. The specific details of the game are of secondary importance for our present purposes. What matters here is the duality of each piece: At each move, the piece can behave as one or the other of its two "states".

2.2.2 Variant 2

A board is created where each of the two players has as many physical Chess pieces as there are possible superpositions, that is, 16×15 pieces in total for each player as described in Section 2.2.1. When selected to move, each individual piece is capable of behaving as one or the other of its two constituent states.

2.2.3 Variant 3

In this variant of the setup in Section 2.2.1, multiple superpositions are allowed. Thus 3-piece superpositions (involving, for example, R_l, B_r, and N_r), 4-piece superpositions, and so on, become part of the game. Each piece can select to behave as one of its constituent pieces whenever it is to make a move.

2.2.4 Variant 4

A larger game may be obtained by combining the variants in Sections 2.2.2 and 2.2.3.

2.2.5 Variant 5

Using the Copenhagen interpretation of quantum physics, we introduce probabilities associated with the choices that a piece has when making a move. Suppose that a piece is a superposition N_l and R_r, then this is denoted as:

$$|\psi\rangle = \alpha|N_l\rangle + \beta|R_r\rangle,$$

where α and β are complex numbers, such that $|\alpha|^2 + |\beta|^2 = 1$. When the piece $|\psi\rangle$ is touched by a player in order to make a move, the piece moves as an N_l with probability $|\alpha|^2$ and as an R_r with probability $|\beta|^2$.

2.2.6 Variant 6

Finally, we may wish to adopt instead the Many Worlds interpretation of quantum physics: Rather than making a choice, a piece makes all possible moves in parallel.

2.3 Something Missing

While the preceding list of variants may be an interesting starting point, offering a new look at Chess, and different perspectives to explore, there is something unsatisfactory about all of them. Indeed, the reader will agree, all variants are quantum by name only. None of them captures the true physical properties of quantum physics. Instead, they "simulate" these properties, and badly at that. One problem is that each piece retains its properties throughout the game. For example, a superposition is never lost. This is not how things are in the quantum world. What we need is Quantum Chess in the real sense.

3 QUANTUM CHESS

" He often played chess with the wise men of his court,
and it was he who gave the pieces human and animal shapes,
assigned them grades and ranks,
and made the king the one who rules all the other pieces."
Mas'ūdī, *The Meadows of Gold*

The variant of Chess that we propose here is *quantum*, as both its pieces and the board on which the game is played satisfy the properties of quantum physics.

3.1 A Quantum Chess Board

We consider an 8×8 Chess board in which the white squares are the same as in a conventional Chess board. The black squares, however, are such that each contains a quantum circuit.

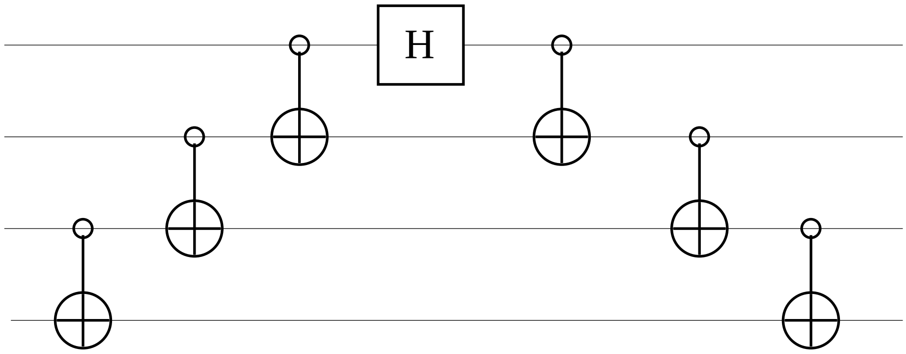

Figure 1: Quantum circuit for black squares on chess board.

Specifically, each black square is a quantum circuit consisting of quantum gates, such as Hadamard gates and Controlled NOT gates. An example of such a circuit was proposed in [12] (see also [4, 13]) and is shown in Fig. 1.

Recall here that:

1. A Hadamard gate **H** operates as follows on the base states $|0\rangle$ and $|1\rangle$ and their superpositions:
$$\mathbf{H}|1\rangle = \frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle$$
$$\mathbf{H}|0\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$$
$$\mathbf{H}(\frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle) = |1\rangle$$
$$\mathbf{H}(\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle) = |0\rangle.$$

2. A Controlled NOT gate operates on two qubits $|x\rangle$ and $|y\rangle$, flipping the second if and only if the first is 1.

The circuit of Fig. 1 has four input lines, four output lines, one Hadamard gate, and six Controlled NOT gates (in the latter, qubit $|x\rangle$ is the top input and qubit $|y\rangle$ the bottom one). The circuit is capable of transforming a quantum superposition of two entangled states to a classical number from 0 to 15, and vice versa, as follows:

$$\begin{aligned}
\frac{1}{\sqrt{2}}(|0000\rangle + |1111\rangle) &\longleftrightarrow |0000\rangle, \\
\frac{1}{\sqrt{2}}(|0000\rangle - |1111\rangle) &\longleftrightarrow |1111\rangle, \\
\frac{1}{\sqrt{2}}(|0001\rangle + |1110\rangle) &\longleftrightarrow |0001\rangle, \\
\frac{1}{\sqrt{2}}(|0001\rangle - |1110\rangle) &\longleftrightarrow |1110\rangle, \\
&\vdots \\
\frac{1}{\sqrt{2}}(|0111\rangle + |1000\rangle) &\longleftrightarrow |0111\rangle, \\
\frac{1}{\sqrt{2}}(|0111\rangle - |1000\rangle) &\longleftrightarrow |1000\rangle.
\end{aligned}$$

3.2 Quantum Chess Pieces

The behavior of the quantum Chess pieces is defined as follows:

1. As before, each quantum Chess piece is in a superposition of states. Since there are 16 pieces for each side then (ignoring color for simplicity) four qubits suffice to represent each piece distinctly. Again for simplicity, we assume henceforth that each quantum Chess piece is a superposition of *two* conventional Chess pieces.

2. "Touching" a piece is tantamount to an "observation"; this collapses the superposition to one of the classical states, and this defines the move that this piece makes.

3. A piece that lands on a white square remains in the classical state in which it arrived. By contrast, a piece that lands on a black square is considered to have traversed a quantum circuit, thereby undergoing a quantum transformation: Having arrived in a classical state, it recovers its original quantum superposition.

4. Initially, each piece on the board is in a quantum superposition of states. However, neither of the two players knows the states in superposition for any given piece.

5. At any moment during the game, the *locations* on the board of all the pieces are known. Each of the two players can see (observe) the states of all the pieces in classical state, but not the ones in superposition.

6. If a player likes the current classical state of a piece about to be moved, then he/she will attempt *not* to land on a quantum circuit. In the opposite case, the player may wish to take a chance by landing the piece on a quantum circuit.

3.3 Variants

Many variations on the Quantum Chess game described above are possible. Two of these are described briefly in what follows.

3.3.1 Variant 1

It is possible to view each quantum Chess piece as a register of 4 independent qubits $q_1q_2q_3q_4$, each in a superposition of $|0\rangle$ and $|1\rangle$. When a piece is 'touched', each qubit collapses (independently of all other qubits and with independent probabilities) to a classical 0 or 1. This yields a classical register of 4 classical bits representing one of the 16 conventional Chess pieces.The latter makes the move according to its definition. For example, if quantum Chess piece $q_1q_2q_3q_4$ collapses to the four classical bits 0100, and the latter was defined as a Knight, then the move is a Knight's move.

3.3.2 Variant 2

The setup here is the same as in Variant 1, with the additional property that a player's pieces are entangled among themselves and with the opponent's pieces. Touching one piece affects another one of the player's own pieces, as well as an opponent's piece.

In both of the above variants it remains the case that a moved piece landing on a white square keeps its conventional status, whereas one that lands on a black square undergoes a transformation making it into a quantum piece. The circuit at each black square may simply be made up of 4 separate Hadamard gates, each operating on a distinct bit of the register.

3.4 Rules

As before, the specific rules of the game are of little importance, and can be defined in many ways that adapt to the new conditions brought about by the properties of quantum physics.

It may be worth mentioning that in all variants presented in the previous two sections, the rules of the game may be adjusted to allow for (or prevent) some unusual situations not encountered in conventional Chess, but quite possible due to the probabilistic nature of Quantum Chess, including:

1. The presence of several pieces of the same type on the board (for example, more than one King of a given color),

2. The absence of a King of a given color at some point in the game.

3.5 To Simulate or Not to Simulate?

There is a tendency in computer science to assume that every computation that is possible on one machine can be simulated on a "universal computer". While this is certainly not true in general [1, 2, 3], the success of the computer as a ubiquitous workhorse in today's society relies to a great extent on the veracity of this principle for most of conventional computations. It is therefore interesting to ask in the present context whether the Quantum Chess Board of Section 3.1 can be simulated on a conventional computer.

The answer here is a clear No. For a simulation, though perhaps able to 'fake' the behavior of the Quantum Chess Board, would certainly defeat the purpose. It is important to remember that in the setup of Section 3.2 each of the two players is given 16 unknown pieces. In the basic case where each piece is a superposition of two conventional pieces, the pieces are each of the form:

$$|\psi\rangle = \alpha|x\rangle + \beta|y\rangle,$$

where $x, y \in \{P_1, P_2, \ldots, R_r\}$, and α and β are *true* random variables. Since none of x, y, α and β is known in advance, a simulation on a conventional computer would necessarily be a fraud. The same applies to the two variants described in Section 3.3. Therefore, only a Quantum Chess Board would be able to render faithfully the relevant properties of quantum physics.

3.6 Some Open Questions

The Quantum Chess Board by its very stochastic nature, presents us with a number of unique questions.

1. Assuming that the goal is to eliminate your opponent's King(s), or capture all of your opponent's pieces, what is the best playing strategy?

2. Assuming that a conventional computer is connected through a suitable interface to the Quantum Chess Board, can such a computer always defeat a human?

3. How would a quantum computer fare against a human?

4. Does a human playing a game of Chess on a Quantum Chess Board have free will, in the sense of being able to control his/her destiny?

5. What about the Quantum Chess pieces, do they have free will [6]?

These are difficult questions. We address one of them in the following section.

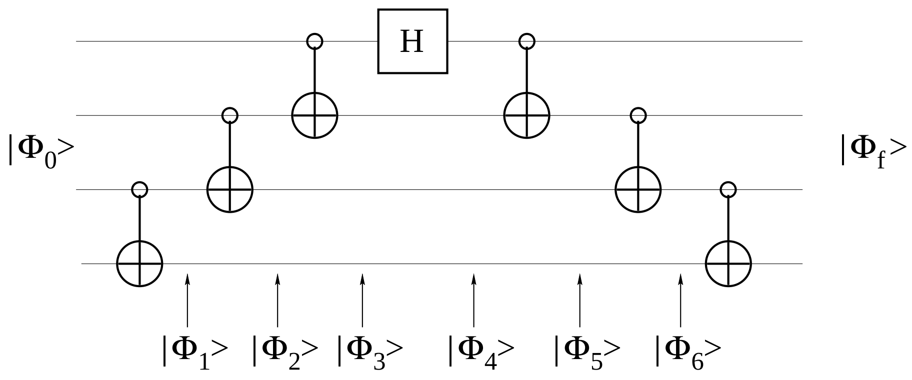

Figure 2: Evolving a Quantum Chess piece through a quantum circuit.

4 WHAT ABOUT A QUANTUM COMPUTER?

"He also made this game an allegory of the heavenly bodies,
that is to say, of the seven planets and the signs of the zodiac."
Mas'ūdī, *The Meadows of Gold*

A quantum computer can certainly play the game of Chess on the Quantum Chess Board as described in Section 3.1. More importantly, it can determine the superposition of each piece *without* observing it in the conventional sense. It does so by *evolving* it through a quantum circuit, such as the one shown in Fig. 1. This is illustrated by the example in Fig. 2 [12], where for the particular input:

$$|\Phi_0\rangle = \frac{1}{\sqrt{2}}|0000\rangle + \frac{1}{\sqrt{2}}|1111\rangle,$$

the intermediate quantum states are:

$|\Phi_1\rangle = \frac{1}{\sqrt{2}}|0000\rangle + \frac{1}{\sqrt{2}}|1110\rangle,$

$|\Phi_2\rangle = \frac{1}{\sqrt{2}}|0000\rangle + \frac{1}{\sqrt{2}}|1100\rangle,$

$|\Phi_3\rangle = \frac{1}{\sqrt{2}}|0000\rangle + \frac{1}{\sqrt{2}}|1000\rangle = (\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle) \otimes |000\rangle,$

$|\Phi_4\rangle = |\Phi_5\rangle = |\Phi_6\rangle = |\Phi_f\rangle = |0000\rangle.$

This allows the unknown superposition to be mapped to a classical integer and then, using table lookup, to determine which two states were superposed. Following this, the initial superposition can be restored by evolving the classical integer backwards through the circuit.

The above discussion tells us that a quantum computer has an apparent edge over a human as well as over a conventional computer: It can determine the superposition hidden in a Quantum Chess piece. This, in turn, leads to an interesting new question: Does this extra knowledge help the quantum computer develop a winning strategy?

5 CONCLUSION

"He consecrated each piece to a star and made it the guardian of the kingdom. When one of their enemies employed a ruse of war against them, they consulted the chessboard to see from which point they would sooner or later be attacked."
Mas'ūdī, *The Meadows of Gold*

Quantum Chess brings to the forefront the importance of 'being quantum'. Indeed, quantum physics appears to be the 'great equalizer' in game playing in general (with Chess as just one example). It opens up a whole new dimension in the discussion about natural versus artificial intelligence. As demonstrated by Quantum Chess, introducing the element of the unknown appears to give humans an equal chance when playing computers.

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