CISC 203 - Assignment 4 (Fall 2019)
Due: Thursday November 28 **by 2:30 PM**
(in the locked CISC 203 drop-off box on Goodwin 2nd floor)

One bonus mark for neatly written student information: Papers that have your name, student number and other information written exactly as requested in the regulations (found at the end), will receive one bonus mark.

1. (2 marks) A person deposits \$1000 in a savings account that yields $x\%$ interest compounded annually ($x \in \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$) and the account charges a fee of y dollars that is deducted from the account at the end of each year. (At the end of the year, first the bank adds the interest and after that deducts the fee.)
   To avoid complications involving accounts with negative balances we assume that the interest paid is greater than the fee, that is, $0.0x \cdot 1000 > y$.

   (a) Let A_n be the money in the account after n years. The initial value A_0 is 1000. Give a recurrence relation for A_n.

   (b) Using the iteration method for first order recurrences solve the recurrence for A_n (that is, give a formula for A_n as a function of n).

2. (3 marks) A string that contains only 0s, 1s and 2s is called a *ternary string.* For $n \geq 0$, let t_n denote the number of ternary strings of length n that do not contain two consecutive 0s.

   Give a recurrence relation for t_n. Please remember to justify your answer! Also give initial values for t_0 and t_1. Using the initial values and the recurrence calculate value of t_5.

3. (3 marks) Using the characteristic root method find a solution for the recurrence

$$b_n = -2b_{n-1} - b_{n-2}$$

   with initial values $b_0 = 5$ and $b_1 = 1$. That is, give a closed form expression for b_n as a function of n (this includes finding the values of the associated constants α_1 and α_2).

4. (3 marks) Using the characteristic root method find a solution for the recurrence

$$a_n = 2a_{n-1} + 2a_{n-2}$$

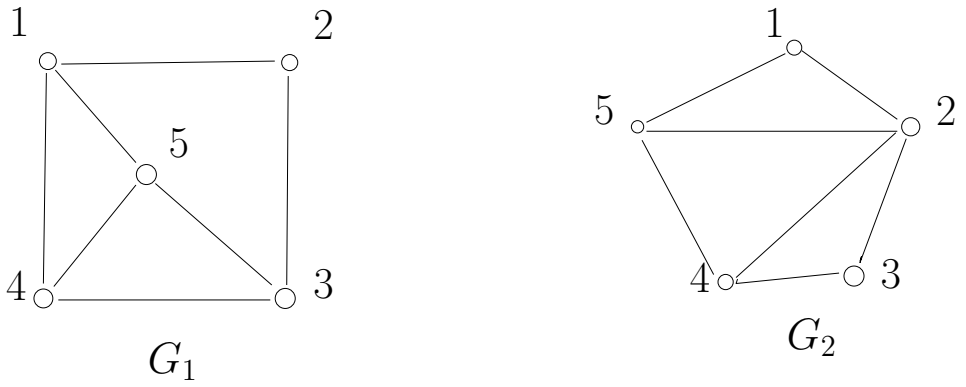

Figure 1: Graphs G_1 and G_2

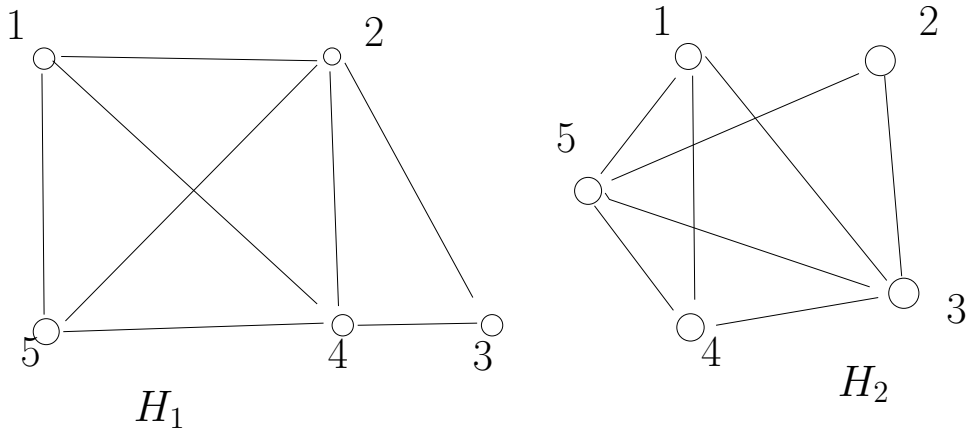

Figure 2: Graphs H_1 and H_2

with initial values $a_0 = 1$ and $a_1 = 3$. That is, give a closed form expression for a_n as a function of n (this includes finding the values of the associated constants α_1 and α_2).

5. (a) (1 mark) Give adjacency matrices for the graphs G_1 and G_2 given in Figure 1.

   (b) (2 marks) Are the graphs G_1 and G_2 given in Figure 1 isomorphic? Either give an explicit isomorphism between G_1 and G_2 or give a rigorous argument showing that an isomorphism does not exist.

   (c) (2 marks) Are the graphs H_1 and H_2 given in Figure 2 isomorphic? Either give an explicit isomorphism between H_1 and H_2 or give a rigorous argument showing that an isomorphism does not exist.

   Note: In (b) and (c) if you answer "yes", it is sufficient to give an explicit isomorphism and you do not need to justify that it is an isomorphism.

6. (2 marks) Consider the graph H_3 given in Figure 3.

   (a) Determine whether H_3 has an Eulerian circuit and construct the circuit if one exists. Justify your answer.

   (b) Determine whether H_3 has an Eulerian trail and construct the trail if one exists. Justify your answer.

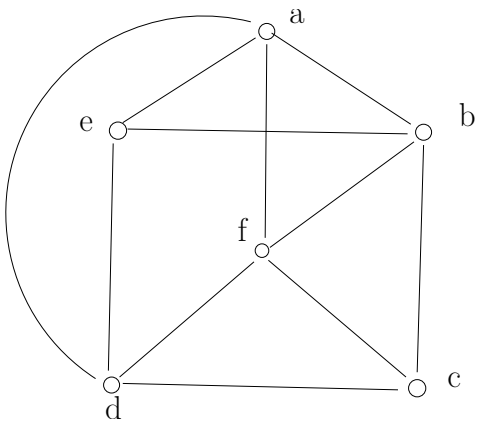

Figure 3: Graph H_3

7. (2 marks) Consider the graph H_4 given in Figure 4.

   (a) Determine whether H_4 has a Hamiltonian cycle and construct the Hamiltonian cycle if one exists. If H_4 does not have a Hamiltonian cycle give a rigorous argument to show why no such cycle exists.

   (b) Determine whether H_4 has a Hamiltonian path and construct the Hamiltonian path if one exists. If H_4 does not have a Hamiltonian path give a rigorous argument to show why no such path exists.

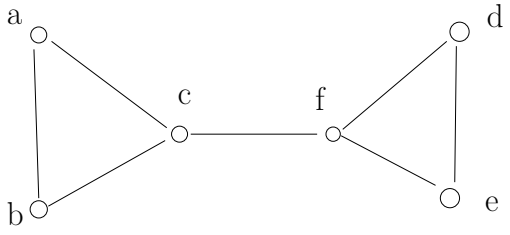

Figure 4: Graph H_4

Regulations on assignments

- **The assignment must be based on individual work.** Copying solutions from other students is a violation of academic integrity. See the course web site for more information `http://research.cs.queensu.ca/home/cisc203/index.html`
- At the top of the first page of the assignment, type or write in clear capital letters the following information on four different lines and in the order listed below:
  - CISC-203 ASSIGNMENT X (where $X \in \{1, 2, 3, 4\}$ is the number of the current assignment)
  - LAST-NAME, FIRST-NAME (your name as it appears on solus, e.g., "SMITH, NANCY")
  - your student number (e.g., "1234 4321")
  - your signature (the signature need not be easily readable)
- **Bonus mark:** Papers that have the above information written exactly correctly and perfectly clearly and legibly will receive one bonus mark. An additional requirement is that it is not permitted to submit more than one copy of the same assignment.
  The assignment is worth 20 marks. Papers that receive the bonus mark, may get more than 20 marks. For the bonus mark there is no partial credit for incomplete information or unclear handwriting.
- The assignment should be put into the locked CISC 203 drop-off box on the 2nd floor of Goodwin hall by the due date. The assignments must be submitted in hardcopy. Assignments sent by email are not accepted.
- If the submission consists of more than one page, the pages must be stapled together.
- *Note:* You are asked to write your solutions using non-erasable pen (or to type the solutions). Solutions written in pencil or erasable ink will be marked, but they will not be considered for remarking after the assignments are returned.