1. **Solution.** By Theorem 2.1 (in the notes on combinatorics) the number of 5-permutations of a nine element set is
$$P(9,5) = \frac{9!}{(9-5)!} = 15120.$$

2. **Solution.** Following the hint, we begin with (b): There are $\binom{40}{20}$ ways to select 20 players for Kingston Frontenacs and this selection uniquely determines also the players for London Knights. The answer is: $\binom{40}{20}$.

   (a): Now the names of the teams have not been specified. This means that selecting a particular 20 people denoted as set X gives exactly the same two teams as selecting the 20 people not belonging to set X. Thus, we should divide the answer from (b) by 2 and get:
$$\binom{40}{20}/2 = \frac{40!}{(20!)^2 \cdot 2}.$$

3. **Solution.**

   (a) Since ABC must be one block, the possible permutations rearrange the block ABC and the individual letters D, E, F, G. The number of permutations of 5 elements is $5! = 120$.

   (b) Now DEBC must be one block and the possible permutations rearrange this block and the letters A, F, G. The number of permutations is $4! = 24$.

   (c) Both strings AB and DC must appear as a block. The possible permutations rearrange these blocks and the three remaining letters. Number of permutations is $5! = 120$.

   (d) Both strings AB and BC occur in the sequence iff the sequence contains ABC. The number of permutations was calculated in case a): 120.

   (e) If AC and DC were to appear in the sequence, the letter C would need to be immediately preceded by both A and D which is impossible. The number of permutations is 0.

   (f) The permutations rearrange blocks CBA and EFG with the letter D. The number of permutations is $3! = 6$.

4. **Solution.**
$$\binom{n+1}{m} = \frac{(n+1)!}{m!(n+1-m)!} = \frac{n+1}{m} \cdot \frac{n!}{(m-1)!(n-(m-1))!} = \frac{n+1}{m} \cdot \binom{n}{m-1}.$$

5. **Solution.**

(a) Choosing the positions of the three 1's completely determines the bit string because there are only two bits. The number of ways to choose 3 positions out of 12 is

$$\binom{12}{3} = \frac{12 \cdot 11 \cdot 10}{3!} = 220.$$

(b) The number of ways to choose at most 3 positions out of 12 is

$$\binom{12}{3} + \binom{12}{2} + \binom{12}{1} + \binom{12}{0} = 220 + 66 + 12 + 1 = 299.$$

(c) The number of ways to choose at least 3 positions out of 12 is

$$\binom{12}{3} + \binom{12}{4} + \binom{12}{5} + \binom{12}{6} + \binom{12}{7} + \binom{12}{8} + \binom{12}{9} + \binom{12}{10} + \binom{12}{11} + \binom{12}{12}$$
$$= 220 + 495 + 792 + 924 + 792 + 495 + 220 + 66 + 12 + 1 = 4017.$$

(The calculation is simplified by recalling that $\binom{n}{k} = \binom{n}{n-k}$.)

(d) As calculated above the number of ways to select 6 positions out of 12 is

$$\binom{12}{6} = 924.$$

6. **Solution.**

(a) The customer can select each variety more than once. The number of choices is the number of 6-combinations of a set with 8 elements with repetition, that is,

$$\frac{(6+8-1)!}{6!(8-1)!} = \frac{13 \cdot 12 \cdot 11 \cdot 10 \cdot 9 \cdot 8}{6!} = 1716$$

(b) Similarly to (a) above, now the number of choices is the number of 12-combinations of a set with 8 elements with repetition, that is,

$$\frac{(12+8-1)!}{12! \cdot 7!} = \frac{19 \cdot 18 \cdot 17 \cdot 16 \cdot 15 \cdot 14 \cdot 13}{7!} = 50388$$

(c) Now the number of choices is the number of 24 combinations of a set with 8 elements with repetition, that is,

$$\frac{(24+8-1)!}{24! \cdot 7!} = \frac{31 \cdot 30 \cdot 29 \cdot 28 \cdot 27 \cdot 26 \cdot 25}{7!} = 2629575$$

(d) We are required to take at least one donut of each kind, that is, the first 8 choices are fixed and only the last 4 donuts can be freely chosen.

The number of choices is the number of 4 combinations of a set with 8 elements with repetition, that is,

$$\frac{(4+8-1)!}{4! \cdot 7!} = \frac{11 \cdot 10 \cdot 9 \cdot 8}{4!} = 330$$

7. **Solution.** Let the $n+1$ integers be $a_1, \ldots, a_{n+1}$. For $j = 1, \ldots, n+1$, write $a_j = 2^{k_j} \cdot b_j$, where $k_j \geq 0$ and b_j is odd.

The integers $b_1, \ldots, b_{n+1}$ are all odd positive integers not exceeding $2n$. Since there are only n odd integers not exceeding $2n$ it follows from the pigeon-hold principle that there exist $1 \leq i < \ell \leq n+1$ such that $b_i = b_\ell$, denote this common value by b.

Now $a_i = 2^{k_i} \cdot b$ and $a_\ell = 2^{k_\ell} \cdot b$. Since $k_i \leq k_\ell$ or $k_\ell \leq k_i$ either a_i divides a_ℓ or vice versa.