1. **Solution.** The total number of 5 card poker hands is $\binom{52}{5}$.

   Next calculate the number of hands with exactly two kings. There are $\binom{4}{2} = 6$ possibilities to choose the two kings. The remaining 3 cards can be chosen among the 48 cards that are not kings.

   Using the product rule the number of hands with exactly two kings is

   $$6 \cdot \binom{48}{3}.$$

   The required probability is then $\frac{6 \cdot \binom{48}{3}}{\binom{52}{5}}$.

2. **Solution.** Below is one counter-example (there are many others).

   $P(Y_1 = 2 \text{ and } Y_2 = 17) = 0$ because $Y_1 = 2$ means that both dice have come up as 1 and then Y_2 gets value 2. On the other hand $P(Y_1 = 2) = P(Y_2 = 17) = \frac{1}{36}$. Note that $Y_1 = 2$ when both dice come up as 1 and $Y_2 = 17$ exactly when the first die comes up as 6 and the second die as 1. Hence $P(Y_1 = 2) \cdot P(Y_2 = 17) \neq 0$.

3. **Solution.** Using Proposition 34.19 and Proposition 34.7:

   $$V(X+Y) = E((X+Y)^2) - E(X+Y)^2 = E(X^2 + 2XY + Y^2) - ((E(X) + E(Y))^2$$
   $$= E(X^2) + 2E(XY) + E(Y^2) - E(X)^2 - 2E(X)E(Y) - E(Y)^2$$

   Since X and Y are independent, by Theorem 34.14, the above simplifies to

   $$E(X^2) + E(Y^2) - E(X)^2 - E(Y)^2 = V(X) + V(Y),$$

   where the last equality again uses Proposition 34.19.

4. **Solution.** Let X_i, $1 \leq i \leq 10$, be a zero-one valued random variable that has value one if the ith coin toss is heads, and zero otherwise. Then $X = X_1 + \ldots + X_{10}$ is the sum of the variables X_i.

   The variables X_i and X_j, $1 \leq i < j \leq 10$, are independent because the coin tosses are independent. Formally this is verified as follows: Let S be a sample space of all possible sequences of 10 coin tosses. For any $s \in S$ and $r_1, r_2 \in \{0, 1\}$,

   $$P(X_i(s) = r_1 \text{ and } X_j(s) = r_2) = \frac{1}{4} = P(X_i(s) = r_1) \cdot P(X_j(s) = r_2).$$

   Let S be the sample space of all possible 2^{10} results of the coin tosses. The expected value of X_i, $1 \leq i \leq 10$, is

   $$E(X_i) = \sum_{s \in S} 2^{-10} X_i(s) = \frac{1}{2}$$

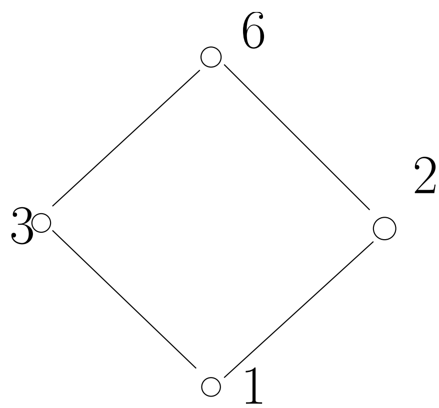

Figure 1: Hasse diagram for question 5 (a)

because exactly in one half of the sequences of coin tosses the ith toss is heads. For the same reason $E(X_i^2) = \frac{1}{2}$ and hence

$$V(X_i) = E(X_i^2) - E(X_i)^2 = \frac{1}{2} - \frac{1}{4} = \frac{1}{4}.$$

Now, since the variables X_i are independent, using the result of question 3 we have

$$V(X) = \sum_{i=1}^{10} V(X_i) = 10 \cdot \frac{1}{4} = \frac{5}{2}.$$

Note that the equality from question 3 extends inductively for sums of more than two pairwise independent variables.

5. **Solution.** The solution is given in Figures 1, 2 and 3.

6. **Solution.**

   (a) If p is a prime, then only p divides p (since $1 \notin A$). Hence all prime numbers are minimal. Furthermore, if $b \in A$ is not a prime then there exists $c \in A$, $c \neq b$, such that c divides b. Therefore the only minimal elements are the prime numbers.

   (b) There are no maximal elements since, for every $b \in A$, b divides $2b \in A$.

7. **Solution.**

   (a) Base case: $\varepsilon^R = \varepsilon$.
   Inductive step: If $w \in \Sigma^*$ and $b \in \Sigma$, then $(wb)^R = bw^R$.

   (b) We use structural induction based on the length of the second string. For a string $y \in \Sigma^*$, denote by $P(y)$ the statement that

$$(x \cdot y)^R = y^R \cdot x^R \text{ for all strings } x$$

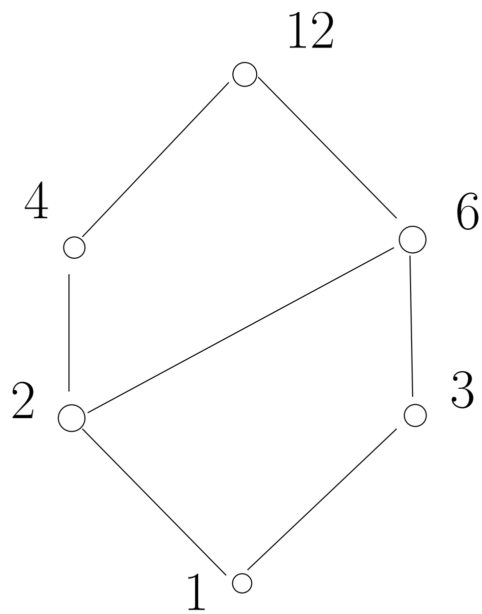

Figure 2: Hasse diagram for question 5 (b)

We prove $P(y)$ using structural induction.
Base case $y = \varepsilon$: $(x \cdot \varepsilon)^R = x^R = \varepsilon^R \cdot x^R$ (because $\varepsilon^R = \varepsilon$)
Inductive step: consider a string yb where $b \in \Sigma$ and $P(y)$ holds. Now for any string x:

$$(x \cdot (yb))^R = ((x \cdot y) \cdot b)^R = b(x \cdot y)^R,$$

where the first equality uses associativity of concatenation and the second equality uses the inductive definition of reversal. Then using the fact that $P(y)$ holds we get

$$b(x \cdot y)^R = b(y^R \cdot x^R) = (by^R) \cdot x^R = (yb)^R \cdot x^R,$$

where the second equality uses associativity and the third equality uses the inductive definition of reversal.
This establishes that $P(yb)$ holds.

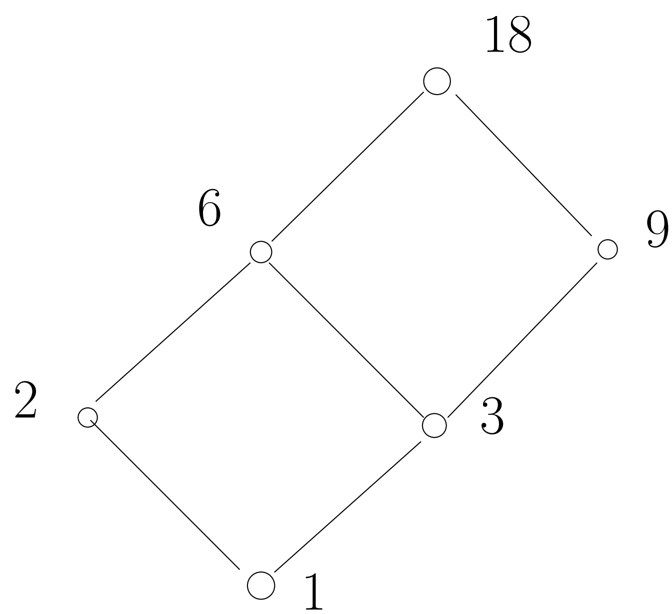

Figure 3: Hasse diagram for question 5 (c)