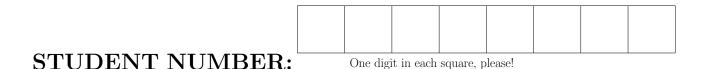
Queen's University CISC-203 Practice Midterm 2019

INSTRUCTIONS

- You have 40 minutes. Attempt all four questions.
- You may bring in one 8.5×11 sheet of paper containing notes, and use it during the midterm.
- Answer each question in the space provided (on the question paper). There is an extra page at the end of the exam if more space is needed. Please write legibly.
- Note: In questions dealing with counting, combinatorics or probability it is not expected that you should compute large numerical values: it is fine to give the final answer in a form like $\binom{56!}{29!}$ or $\binom{40}{21}$ as long as you clearly explain how you arrived at the answer.
- Please note: You are asked to write your answers using a *non-erasable pen*. Answers written in pencil or erasable ink will be marked, but they will not be considered for remarking after the midterms are returned.



Student number (written in words):

MARKS

Problem 1	/X
Problem 2	/Y
Problem 3	/Z
Problem 4	/W
Total	/X + Y + Z + W

 ${\tt Code:} \quad {\tt askjkhkhdhfkjhqwewe1yu2tuytuyue1tduayt4duytu12yqeqwe}$

Student #

1. Consider relations $R \subseteq A \times B$ and $S \subseteq B \times C$. Prove that

$$(R \odot S)^{-1} = S^{-1} \odot R^{-1}.$$

Here $R \odot S$ denotes the composition of relations R and S.

- 2. A three-digit natural number is a number of the form $d_1d_2d_3$, where d_1 , d_2 , and d_3 are elements from the set $\{0, 1, \ldots, 9\}$ and $d_1 \neq 0$. In other words, a three-digit natural number is any number between 100 and 999.
 - (i) How many three-digit natural numbers can be formed if each digit must be distinct?
 - (ii) How many three-digit natural numbers can be formed if the number is even and digits may be repeated? (A number is even if the digit d_3 is an element from the subset $\{0, 2, 4, 6, 8\}$.)
 - (iii) How many three-digit natural numbers can be formed if the number is even, digits may be repeated, and the number contains the digit 5?

- 3. How many functions from the set $\{1, 2, ..., n\}$ to $\{0, 1\}$ $(n \ge 1)$ there exist
 - (i) that are one-to-one?
 - (ii) that map both 1 and n to 0?
 - (iii) that map exactly one of the integers $1, \ldots n-1$ to 1?

Justify your answers!

Note: In some of the cases one may need to consider separately a few small values of n.

4. A standard deck of playing cards contains 52 cards. Each card has a rank from the set $\{2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K, A\}$ and a suit from the set $\{\clubsuit, \diamondsuit, \heartsuit, \clubsuit\}$.

Assume that the ranks J, Q, and K have numerical values of 10 and assume that the rank A has a numerical value of 11.

For the following questions, let X be a random variable corresponding to the numerical value of the first card drawn from a shuffled deck and let Y be a random variable corresponding to the numerical value of the second card drawn from the same shuffled deck.

Below P(e) stands for probability of event e and E(X) is the expected value of random variable X.

- (i) What is P(X is even)?
- (ii) What is E(X)?
- (iii) Are X and Y independent? Explain why or why not.
- (iv) Using your answer from part (ii), determine E(X + Y).

Student #

(Extra page)