Queen's University

CISC-203 Practice Midterm 2019

INSTRUCTIONS

- You have 40 minutes. Attempt all four questions.
- You may bring in one 8.5×11 sheet of paper containing notes, and use it during the midterm.
- **Answer each question in the space provided** (on the question paper). There is an extra page at the end of the exam if more space is needed. **Please write legibly.**
- *Note:* In questions dealing with counting, combinatorics or probability it is not expected that you should compute large numerical values: it is fine to give the final answer in a form like "$\frac{56!}{29!}$" or "$\binom{40}{21}$" as long as you clearly explain how you arrived at the answer.
- **Please note:** You are asked to write your answers using a *non-erasable pen.* Answers written in pencil or erasable ink will be marked, but they will not be considered for remarking after the midterms are returned.

STUDENT NUMBER: One digit in each square, please!

Student number (written in words):

MARKS

Problem 1	/X
Problem 2	/Y
Problem 3	/Z
Problem 4	/W
Total	/X + Y + Z + W

Code: askjkhkhdhfkjhqwewe1yu2tuytuyue1tduayt4duytu12yqeqwe

1. Consider relations $R \subseteq A \times B$ and $S \subseteq B \times C$. Prove that

$$(R \odot S)^{-1} = S^{-1} \odot R^{-1}.$$

Here $R \odot S$ denotes the composition of relations R and S.

2. A three-digit natural number is a number of the form $d_1 d_2 d_3$, where d_1, d_2, and d_3 are elements from the set $\{0, 1, \ldots, 9\}$ and $d_1 \neq 0$. In other words, a three-digit natural number is any number between 100 and 999.

   (i) How many three-digit natural numbers can be formed if each digit must be distinct?

   (ii) How many three-digit natural numbers can be formed if the number is even and digits may be repeated? (A number is even if the digit d_3 is an element from the subset $\{0, 2, 4, 6, 8\}$.)

   (iii) How many three-digit natural numbers can be formed if the number is even, digits may be repeated, and the number contains the digit 5?

3. How many functions from the set $\{1, 2, \ldots, n\}$ to $\{0, 1\}$ $(n \geq 1)$ there exist

   (i) that are one-to-one?

   (ii) that map both 1 and n to 0?

   (iii) that map exactly one of the integers $1, \ldots n-1$ to 1?

   Justify your answers!
   Note: In some of the cases one may need to consider separately a few small values of n.

4. A standard deck of playing cards contains 52 cards. Each card has a rank from the set $\{2, 3, 4, 5, 6, 7, 8, 9, 10, \text{J}, \text{Q}, \text{K}, \text{A}\}$ and a suit from the set $\{\clubsuit, \diamondsuit, \heartsuit, \spadesuit\}$.

   Assume that the ranks J, Q, and K have numerical values of 10 and assume that the rank A has a numerical value of 11.

   For the following questions, let X be a random variable corresponding to the numerical value of the first card drawn from a shuffled deck and let Y be a random variable corresponding to the numerical value of the second card drawn from the same shuffled deck.

   Below $P(e)$ stands for probability of event e and $E(X)$ is the expected value of random variable X.

   (i) What is $P(X \text{ is even})$?

   (ii) What is $E(X)$?

   (iii) Are X and Y independent? Explain why or why not.

   (iv) Using your answer from part (ii), determine $E(X + Y)$.

(Extra page)