Lab 4 Continued ...

As I hope you realized, we can reduce the size of the problem by doing n/2 pairwise tests of the chips and putting aside most of the chips. We can put aside all the pairs that reported either one or both of the chips as faulty, and focus on the chip-pairs that reported both chips as good. Each of these pairs consists of either two good chips or two faulty ones, and a bit of algebra* shows that within this set of chips there must be more good chips than bad chips. But this means that if we choose one chip out of each pair, the chosen chips must have more good chips than bad chips. Thus we have put aside at least half the original set of chips and we have created a smaller instance of the original problem – we can recursively (or iteratively) apply the same technique to identify one good chip in the smaller set.

We need to think about what to do when n is odd. In this case we randomly extract one of the chips from the set, apply the pair-test method to the n-1 remaining chips, then reintroduce the extracted chip to the reduced set, ... BUT ONLY if the reduced set has an even number of chips in it. (The reasoning behind this special case is not difficult, but I will leave it as an exercise for you. Basically we show that if the reduced set is odd in size, we definitely don't need to keep the extracted chip, but if the reduced set is even in size, we **may** need the extracted chip, so we put it back in.)

Of course this reduction process ends when we have only one chip left in the reduced set. Since at every stage we are guaranteed that the reduced set contains more good chips than faulty ones, when there is only one chip left it must be good.

Once we have absolutely identified a good chip, we can use that chip to test all the others to positively identify the good ones.

Input for this problem consists of a file of integers. The first line contains a single integer indicating the number of days for which input will be provided. The remaining lines occur in pairs. The first line in each pair indicates the number of chips produced that day. The second line in each pair lists a sequence of 0's and 1's, representing faulty and good chips respectively.

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Let n be the number of chips, let g be the number of good chips, and let f be the number of faulty chips. We know $n = g + f$ and $g > f$. Assume for the sake of simplicity that n is even, so $g - f >= 2$. Suppose that in the initial pairing of chips, there are x pairs of faulty chips. This means that there are $f - 2*x$ faulty chips paired with good chips. This means that there are $(g - (f - 2*x))/2$ pairs of good chips. This last expression simplifies to $(g - f)/2 + x$, which is greater than x since $(g – f)/2$ is at least 1. Thus if we take one chip from each pair that reports "both good" we are guaranteed that in this reduced set there will be more good chips than faulty chips.