CISC/CMPE422, CISC835: Formal Methods in Software Engineering

Juergen Dingel

Fall 2019

Lecture: Predicate Logic

Semantics of Predicate Logic: Examples (pages 41-44 in courseware)

Let

$$\mathcal{F} \stackrel{def}{=} \{zero, succ, add\}$$

where *zero*, *succ*, and *add* have arity 0, 1, and 2 respectively. Also, let

$$\mathcal{P} \stackrel{def}{=} \{\leq, isZero\}$$

where $\leq$ takes two arguments and *isZero* takes one argument.

1. Let the model $\mathcal{M}_1$ be given by
   - $\mathcal{D}^{\mathcal{M}_1} \stackrel{def}{=} \mathbb{N}$,
   - $\mathcal{F}^{\mathcal{M}_1} \stackrel{def}{=} \{zero^{\mathcal{M}_1}, succ^{\mathcal{M}_1}, add^{\mathcal{M}_1}\}$ where $zero^{\mathcal{M}_1} = 0 \in \mathbb{N}$, $succ^{\mathcal{M}_1}$ is the successor function on $\mathbb{N}$ and $add^{\mathcal{M}_1}$ is the addition function on $\mathbb{N}$, and
   - $\mathcal{P}^{\mathcal{M}_1} \stackrel{def}{=} \{\leq^{\mathcal{M}_1}, isZero^{\mathcal{M}_1}\}$ where $\leq^{\mathcal{M}_1}$ is "less than or equal" on $\mathbb{N}$, and $isZero^{\mathcal{M}_1}(x) = true$ if and only if $x = 0$.

Semantics of Predicate Logic: Examples (pages 41-44 in courseware)

Let

$$\mathcal{F} \stackrel{def}{=} \{zero, succ, add\}$$

where *zero*, *succ*, and *add* have arity 0, 1, and 2 respectively. Also, let

$$\mathcal{P} \stackrel{def}{=} \{\leq, isZero\}$$

where $\leq$ takes two arguments and *isZero* takes one argument.

2. Let the model $\mathcal{M}_2$ be given by
   - $\mathcal{D}^{\mathcal{M}_2} \stackrel{def}{=} \{0,1\}^*$,
   - $\mathcal{F}^{\mathcal{M}_2} \stackrel{def}{=} \{zero^{\mathcal{M}_2}, succ^{\mathcal{M}_2}, add^{\mathcal{M}_2}\}$ where $zero^{\mathcal{M}_2}$ is the empty word ϵ, $succ^{\mathcal{M}_2}(w) = w1$ for all $w \in \{0,1\}^*$, and $add^{\mathcal{M}_1}(w_1, w_2) = w_1 w_2$ for all $w_1, w_2 \in \{0,1\}^*$, and
   - $\mathcal{P}^{\mathcal{M}_2} \stackrel{def}{=} \{\leq^{\mathcal{M}_2}, isZero^{\mathcal{M}_2}\}$ where $\leq^{\mathcal{M}_2}$ is the prefix relation on $\{0,1\}^*$, that is, $w_1 \leq^{\mathcal{M}_2} w_2$ if and only if w_1 is a prefix of w_2, and $isZero^{\mathcal{M}_2}$ returns *true* if and only if the argument is ϵ.

Semantics of Predicate Logic: Examples (pages 41-44 in courseware)

3. Let the model $\mathcal{M}_3$ be given by
   - $\mathcal{D}^{\mathcal{M}_3} \stackrel{def}{=} \{off, on\}$,
   - $\mathcal{F}^{\mathcal{M}_3} \stackrel{def}{=} \{zero^{\mathcal{M}_3}, succ^{\mathcal{M}_3}, add^{\mathcal{M}_3}\}$ where $zero^{\mathcal{M}_3} = off$ and

$$\begin{aligned} succ^{\mathcal{M}_3}(off) &= on \\ succ^{\mathcal{M}_3}(on) &= off, \end{aligned}$$

   and $add^{\mathcal{M}_3}$ is the function defined by

$$\begin{aligned} add^{\mathcal{M}_3}(off, off) &= off \\ add^{\mathcal{M}_3}(off, on) &= add^{\mathcal{M}_3}(on, off) = on \\ add^{\mathcal{M}_3}(on, on) &= off, \end{aligned}$$

   and
   - $\mathcal{P}^{\mathcal{M}_3} \stackrel{def}{=} \{\leq^{\mathcal{M}_3}, isZero^{\mathcal{M}_3}\}$ where

$$\leq^{\mathcal{M}_3}(x, y) = \begin{cases} false, & \text{if } x = on \text{ and } y = off \\ true, & \text{otherwise.} \end{cases}$$

   and $isZero^{\mathcal{M}_3}(x) = true$ if and only if $x = off$.