CISC 462 Assignment 4 Postmortem

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1. **Part (a): 5 marks. Part (b): 5 marks.**

   Part (a) was done well by all students.

   Part (b) was generally done well, but some students misinterpreted the question. The question asked about finding a reduction that worked for any non-trivial language B; that is, coming up with a procedure that tells us about strings either in or not in B. We cannot say anything about the complexity class in which B is contained.

   In general, if you start a proof with the phrase "Assume $\mathsf{P} = \mathsf{NP}$", you should first consider if a better approach exists.

2. **Showing DOUBLE-SAT $\in$ NP: 3 marks. Reduction from SAT: 7 marks.**

   Many students forgot to include a proof that DOUBLE-SAT $\in \mathsf{NP}$. Remember, NP-completeness proofs consist of two parts: showing that the problem is in NP, and showing that the problem is NP-hard. The reduction from SAT only constitutes the proof of NP-hardness.

3. **Whole question: 10 marks.**

   The most common (small) error was not properly explaining why $B \in \mathsf{L}$. A sufficient one-line explanation is that if the counter used to keep track of parentheses is represented in binary, then storing a value n would require $\log(n)$ space. However, quite a few students didn't mention that the counter had to be represented in binary for the space requirement to be met.

4. **Showing $A_{LBA} \in$ PSPACE: 3 marks. Showing A_{LBA} is PSPACE-hard: 7 marks.**

   This question was mostly done well. Any major concerns were raised already in the comments for question 2 (just replace DOUBLE-SAT with A_{LBA} and NP with PSPACE).

5. **Part (a): 5 marks. Part (b): 5 marks.**

   Part (a) was done well by most students.

   Part (b) was unfortunately not as generous as part (a). Few students got this part of the question correct, and many incorrect answers were due to faulty reasoning.

   We know that $A_{LBA} \in \mathsf{PSPACE}$ (and, in fact, it is PSPACE-complete). We also know that $\mathsf{P} \subseteq \mathsf{PSPACE}$. But this does not imply that $A_{LBA} \in \mathsf{P}$. (By analogy: let $A = \{1, 3, 5\}$ and $B = \{1, 2, 3, 4, 5, 6\}$. We know that $2 \in B$. We also know that $A \subseteq B$. But $2 \notin A$.)

   An alternate proof assumed that $\mathsf{P} = \mathsf{SPACE}(n)$ and went on to reach a contradiction, supposedly showing that $A_{LBA} \notin \mathsf{P}$. However, such an assumption itself contradicts the Space Hierarchy Theorem and is therefore invalid.

6. **Each part: 2 marks.**

   Marking for this question was straightforward. Half-marks were given if you wrote a non-strict inclusion instead of a strict inclusion for any part.

   The most common problem was trying to apply the Time Hierarchy Theorem to part (e).

Questions/comments? Feel free to stop by my office hours or send me an email at `tsmith [at] cs [dot] queensu [dot] ca`.