Computability and Complexity, CISC 462 - Assignment 3 (Fall 2018, K. Salomaa)
Due in lecture 9:30 AM, Monday November 5

1. (a) Find a match for the following instance of the Post Correspondence Problem (PCP).

$$\{\ [\frac{00}{001}],\ [\frac{11}{10}],\ [\frac{011}{1}]\ \}$$

   (b) Do the below two PCP instances have a match? Justify your answers.

   i. $\{\ [\frac{1}{101}],\ [\frac{01}{10}],\ [\frac{011}{1}]\ \}$

   ii. $\{\ [\frac{01111}{011}],\ [\frac{101}{11}],\ [\frac{100}{1000}],\ [\frac{1010}{110}]\ \}$

2. Consider a PCP instance

$$(u_1, v_1), \ldots, (u_m, v_m),$$

$u_i, v_i \in \Sigma^*$, $i = 1, \ldots, m$. We say that the above instance has a *non-constrained solution* if there exist sequences of integers $i_1, \ldots, i_s$ and $j_1, \ldots, j_r$, $r, s \geq 1$ ($i_x, j_y \in \{1, \ldots, m\}$), such that

$$u_{i_1} \cdots u_{i_s} = v_{j_1} \cdots v_{j_r}$$

Define

$$L_{ncPCP} = \{< P > \mid\ P \text{ is an instance of PCP that has a non-constrained solution } \}$$

Show that L_{ncPCP} is decidable.

3. Recall

$$E_{CFG} = \{< G > \mid G \text{ is a context-free grammar and } L(G) = \emptyset\}.$$

- Answer the following questions and **justify** your answers. ("$\leq_m$" denotes mapping reducibility.)
  In each case where you answer "yes", please give a concrete example of the language and the corresponding mapping reduction.

(a) Does there exist a decidable language A such that $A \leq_m E_{CFG}$?

(b) Does there exist a decidable language B such that $E_{CFG} \leq_m B$?

(c) Does there exist an undecidable language C such that $C \leq_m E_{CFG}$?

(d) Does there exist an undecidable language D such that $E_{CFG} \leq_m D$?

4. Answer each part TRUE or FALSE.

   (a) $n = O(n^2)$
   (b) $n = o(2n)$
   (c) $n^2 = o(n^3)$
   (d) $3^n = 2^{O(n)}$
   (e) $2^{2n} = O(2^n)$
   (f) $2^n = o(2^{n+5})$
   (g) $n^2 = O(n \cdot (\log n)^3)$
   (h) $(\log n)^3 = o(\sqrt{n})$
   (i) $n^2(\log n)^3 = o(n^3)$
   (j) $\frac{1}{n} = o(1)$

5. Define

$$ALL_{\text{DFA}} = \{< A > \mid A \text{ is a DFA and } L(A) = \Sigma^*\}.$$

   Show that $ALL_{\text{DFA}} \in \text{P}$.

6. (a) A *triangle* in an undirected graph is a 3-clique and define

$$\text{TRIANGLE} = \{< G > \mid G \text{ is an undirected graph that contains a triangle }\}.$$

   Show that TRIANGLE $\in$ P.

   (b) Explain why your algorithm solving TRIANGLE in polynomial time cannot be extended to give a polynomial time algorithm for CLIQUE.
   Recall that the language CLIQUE is defined in section 7.3 of the text:

$$\text{CLIQUE} = \{< G, k > \mid G \text{ is an undirected graph with a } k\text{-clique }\}.$$

Regulations on Assignments

- As described on the course homepage, all assignments must be based on *individual work*.
- The assignments are graded according to the correctness, preciseness and elegance of the solutions.
- If, as part of your solution, you rely on results from the textbook you should clearly state which result(s) you are using.
- Each question is worth 10 marks and the assignment is marked out of 60 marks.