Computability and Complexity, CISC 462 - Assignment 4 (Fall 2018, K. Salomaa)
Due in lecture 9:30 AM, Monday November 26

1. (a) Show that the polynomial time reducibility, $\leq_P$, is a transitive relation.
   (b) Prove the following:
   If $A \in$ P then, for any language B that is not $\emptyset$ or Σ^*, we have $A \leq_P B$.

2. Let DOUBLE-SAT $= \{\langle\phi\rangle \mid \phi$ has at least two satisfying assignments $\}$.
   Show that DOUBLE-SAT is NP-complete. (You can assume known that "ordinary" satisfiability SAT is NP-complete.)

3. Let B be the language of properly nested parentheses. For example, (()) and (()(()))() are in B, but)(is not. Show that B is in L $(=$ SPACE$(\log n))$.

4. Define $A_{\text{LBA}} = \{ < M, w > \mid M$ is an LBA that accepts string $w \}$.
   Show that A_{LBA} is PSPACE-complete.

5. Recall that in Question 4 we showed that A_{LBA} is PSPACE-complete.
   (a) Is it known [1] whether or not $A_{\text{LBA}} \in$ NL? Explain your answer.
   (b) Is it known whether or not $A_{\text{LBA}} \in$ P? Explain your answer.

6. What is the relationship (equal, strict inclusion in one direction (which one?), inclusion that is not known to be strict[2]) between the following pairs of complexity classes. **Justify your answers.**
   (a) $NSPACE(n \cdot \log n)$ and $SPACE(n^3)$
   (b) $TIME(n^3 \cdot \log n)$ and $TIME(n^3 \cdot \sqrt{n})$
   (c) $TIME(2^n)$ and $TIME(3^n)$
   (d) $NTIME(n \cdot \sqrt{n})$ and $SPACE(n^3 \cdot \log n)$

[1] Here "known" means "known for sure" or that the claim has been proven.
[2] Here "*not known to be strict*" means "*on the basis of the results in chapters 7, 8 and 9 in our textbook, it is not known to be strict*".

(e) $TIME(f(n))$ and $TIME(n^3)$ where

$$f(n) = \begin{cases} n^5, \text{ when } n \leq 2^{1000}, \\ n^3 \cdot \log n, \text{ otherwise.} \end{cases}$$

Regulations on Assignments

- As described on the course homepage, all assignments must be based on *individual work.*
- The assignments are graded according to the correctness, preciseness and elegance of the solutions.
- If, as part of your solution, you rely on results from the textbook you should clearly state which result(s) you are using.
- Each question is worth 10 marks and the assignment is marked out of 60 marks.