Assignment 3, CISC422/853, Winter 2009
Due: Thursday, March 12, 2009

Question 1: (Formalization, 15 points)

Let Hans and Franz be two users of a shared resource R. Suppose we have the following atomic propositions:

$u.req$	is true iff user u is requesting the resource R
$u.use$	is true iff user u is using the resource R
$u.rel$	is true iff user u is releasing the resource R

where $u \in \{\text{Hans, Franz}\}$. Use LTL to formalize the following requirements:

1. (3 pts) At most one user can use R at any time.
2. (3 pts) A user can use R only for a finite amount of time.
3. (3 pts) If a user wants use R, he/she will eventually be able to do so.
4. (3 pts) A user can always request to use R.
5. (3 pts) Users can only use R in an alternating fashion, i.e., if user u_1 stops using R, then the next user using R must be u_2 (assuming $u_1 \neq u_2$).

Question 2: (LTL, 18 points)

Let ψ_1 and ψ_2 be LTL formulas. Consider the following new temporal operators:

1. "At next" ψ_1 N ψ_2: at the next time where ψ_2 holds, ψ_1 also holds.
2. "While" ψ_1 W ψ_2: ψ_1 holds at least as long as ψ_2 does.
3. "Before" ψ_1 B ψ_2: if ψ_2 holds sometime, ψ_1 does so before.

For each of these new temporal operators,

1. (3 pts) extend the satisfaction relation $r \models \varphi$ for LTL formulas presented in class by an appropriate clause defining precisely the circumstances under which a formula using this new operator (i.e., a formula of the form ψ_1 N ψ_2, ψ_1 W ψ_2, or ψ_1 B ψ_2) holds in some run/execution r of some Buechi Automaton M.
2. (3 pts) express them in terms of the standard LTL operators. In other words, for each of ψ_1 N ψ_2, ψ_1 W ψ_2, or ψ_1 B ψ_2, find an equivalent LTL formula. Make sure that your LTL formula is indeed equivalent to the given formula with respect to your definition of the temporal operator given in the previous question.

Question 3: (CTL, 24 pts)

For each of the following pairs of CTL formulas determine if the two formulas are equivalent. If the two formulas are not equivalent, draw the beginning of a computation tree T that makes one formula true but not the other. Clearly indicate which formula is true in T and which is false. If the two formulas are equivalent, give a brief, informal explanation.

1. (4 pts) $\mathbf{EG}p \vee \mathbf{EG}q$ and $\mathbf{EG}(p \vee q)$
2. (4 pts) $\mathbf{AF}p \vee \mathbf{AF}q$ and $\mathbf{AF}(p \vee q)$
3. (4 pts) *true* and $\mathbf{EG}p \rightarrow \mathbf{AG}p$
4. (4 pts) $\mathbf{A}\big[p\ \mathbf{U}\ q\big]$ and $p \wedge \mathbf{AF}q$
5. (4 pts) $\mathbf{A}\big[p\ \mathbf{U}\ q\big] \vee \mathbf{A}\big[r\ \mathbf{U}\ q\big]$ and $\mathbf{A}\big[p \vee r\ \mathbf{U}\ q\big]$
6. (4 pts) $\mathbf{EX}\ \mathbf{EF}(p \vee q)$ and $\mathbf{AX}\ \mathbf{EF}(p \vee q)$