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Iverson, K.E.; Notation as a Tool of Thought; CACM 23, 8 (August 1980) pp. 444-465.

The paper discusses the importance of notations in general and of the characteristics which make a mathematical notation useful. It identifies a list of the characteristics, which the author thinks are important. Some of the characteristics which are mentioned, are Universality, which is taken to mean the ability of a notation to describe a wide range of problems. Executability, which is described as the ability to execute the notation, and a third characteristic is lack of ambiguity.

Programming languages score well with all three of these characteristics, but unfortunately do not, in general fare as well with the others. The paper then advances APL as an example language which has all of the required characteristics. APL itself is not sufficient, so a small number of additions are made to enhance its usefulness as a notation. An example is a symbol to assert equivalence.

The next section is a discussion of APL used as a mathematical notation. In this section APL is examined in terms of each of the characteristics mentioned earlier. In the process APL itself is being introduced in context, as other mathematical notations generally are. From here, two chapters are spent developing and motivating some results on polynomials and on conversions between representations for a number of different useful items (e.g. graphs, integers). These are not formal proofs, but discussions, using the notation as a tool in the motivation, similar to many mathematics textbooks.

He then follows with a section of formal proofs of a number of identities and lemmas, using APL as notation.

In the concluding section Iverson indicates that he feels that the question of whether a programming language can be useful as a mathematical notation is an important one and believes that it can be. He also says that even if APL is found to be not entirely suitable, that other work should be done.

He compares APL with standard mathematical notations and points out a few ways in which APL is superior. The paper finishes by pointing out that others appear to agree with APL's usefulness as a notation, and in fact references a few books which use APL as a notation to teach a subject.

The paper makes a good point for APL's ability to act as a notation. It is certainly not what we are accustomed to in terms of mathematical notations and at first glance seem to be confusing. This may well be due to lack of familiarity. One question which arises, is whether the difficulty of formal verification of aspects of APL might be a problem. Iverson never mentions this point at all.