**CISC-102**

HOMEWORK 3

## Readings

Read sections 1.8 of *Schaum's Outline of Discrete Mathematics.*
Read section 2.1 of *Discrete Mathematics Elementary and Beyond.*

## Problems

(1) Mathematical induction can be used to prove that the sum of the first $n$ natural numbers is equal to $\frac{n(n+1)}{2}$. This can also be stated as:

We can prove that the proposition $\mathrm{P}(n)$,

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

is true for all $n \in \mathbb{N}$, by using mathematical induction.

I wrote out the proof, but somehow it got all scrambled as shown below. Rearrange the lines to get the correct proof.

1. **Induction step:** The goal is to show that $\mathrm{P}(k+1)$ is true.

2. **Base:** for $n$ =1, $1 = \frac{1(1+1)}{2}$

3. $= \frac{(k+1)(k+2)}{2}$

4. $\sum_{i=1}^{k+1} i = \sum_{i=1}^{k} i + (k+1)$

5. $= \frac{k^2+k+2k+2}{2}$

6. **Induction hypothesis:** Assume that $\mathrm{P}(k)$, for Natural numbers $k \geq 1$ is true, that is:

7. $= \frac{k^2+3k+2}{2}$

8. $\sum_{i=1}^{k} i = \frac{k(k+1)}{2}$

9. $= \frac{k(k+1)}{2} + (k+1)$

(2) Prove using mathematical induction that the proposition P($n$),

$$\sum_{i=2}^{n} i = \frac{(n-1)(n+2)}{2}$$

is true for all $n \in \mathbb{N}, n \geq 2$.

(3) Prove using mathematical induction that the proposition P($n$),

$$\sum_{i=3}^{n} i = \frac{(n-2)(n+3)}{2}$$

is true for all $n \in \mathbb{N}, n \geq 3$.

(4) Prove using mathematical induction that the proposition P($n$)

$$n! \leq n^n$$

is true for all $n \in \mathbb{N}$.

(5) Given a set of $n$ points on a two dimensional plane, such that no three points are on the same line, it is always possible to connect every pair of points with a line segment. The figure illustrates this showing 5 points, that are pairwise connected with 10 line segments. Prove using mathematical induction that the total number of line segments is $\frac{n(n-1)}{2}$ for any number of points $n \in \mathbb{N}, n \geq 2$.

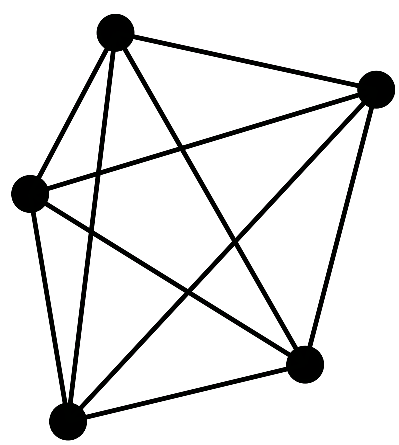

FIGURE 1. Five points, pairwise connected with 10 line segments.

(6) Let $T_n$ denote the number of two element subsets of a set with $n$ elements. Now observe that $T_{n+1} = T_n + n$. Can you explain why this is true.?

(7) Use the result from the previous question and mathematical induction to prove that the number of two element subsets of a set of size $n$ is $\frac{n(n-1)}{2}$.

(8) Consider the following proof that $n + 1 = n$, for all natural numbers $n$.

**Induction Hypothesis**: Assume that $k + 1 = k$ for a fixed natural number $k$.

**Induction step**:

$$\begin{aligned} k + 2 =& k + 1 + 1 \\ =& k + 1 \text{(apply induction hypothesis)} \\ =& k + 1 \end{aligned}$$

We have shown that P($k$) true implies that P($k + 1$) is true so by the principle of mathematical induction we conclude that P($n$) is true for all $n \in \mathbb{N}$. □

This can't possibly be right! What's wrong?