CISC-102 WINTER 2020

HOMEWORK 4

Assignments will **not** be collected for grading.

Read sections 3.1, 3.2, 3.3, 3.4, 3.5, 3.6, (in 3.2 you may skip the part on composition and in 3.3 you may skip the part on permutations) of *Schaum's Outline of Discrete Mathematics*.

Read section 2.1 again (If you did not understand things last week) of *Discrete Mathematics Elementary and Beyond*.

(1) Determine whether the mappings given below where $f : \mathbb{R} \to \mathbb{R}$ are or are not functions, and explain your decision.

   (a) $f(x) = 1/x$
   (b) $f(x) = \sqrt{x}$
   (c) $f(x) = 3x - 3$

(2) Determine whether each of the following functions from $\mathbb{R}$ to $\mathbb{R}$ is a bijection, and explain your decision. HINT: Plotting these functions may help you with your decision.

   (a) $f(x) = 3x + 4$
   (b) $f(x) = -x^2 + 2$
   (c) $f(x) = x^3 - x^2$

(3) Suppose the function $f : A \to B$ is a bijection. What can you say about the values $|A|$ and $|B|$?

(4) Consider the recursive function $T(1) = 1, T(n) = T(n-1) + 1$, for all $n \geq 2$.

   (a) Use the recursive definition to obtain values T(2), T(3), and T(4).
   (b) Using the values that you obtained for T(2), T(3), and T(4), to guess the value of T(n), and then prove that it is correct using induction.

(5) Consider the recursive function $F(1) = 3, F(n) = 3F(n-1)$, for all $n \geq 2$.

(a) Use the recursive definition to obtain values F(2), F(3), and F(4).

(b) Use the values that you obtained for F(2), F(3), and F(4), to guess the value of F(n), and then prove that it is correct using induction.

(6) Consider the recursive function $G(1) = x, G(n) = xG(n-1)$, for all $n \geq 2$

(a) Use the recursive definition to obtain values G(2), G(3), and G(4).

(b) Use the values that you obtained for G(2), G(3), and G(4), to guess the value of G(n), and then prove that it is correct using induction.