CISC-102 WINTER 2020

HOMEWORK 6

Assignments will **not** be collected for grading.

Readings

Read sections 11.1, 11.2, 11.3, 11.4, and 11.5 of *Schaum's Outline of Discrete Mathematics*.

Read section 6.1, and 6.2 of *Discrete Mathematics Elementary and Beyond*.

Problems

(1) Find the quotient q and remainder r, as given by the Division Algorithm theorem for the following examples.
  (a) a = 500, b = 17
  (b) a = -500, b = 17
  (c) a = 500, b = -17
  (d) a = -500, b = -17

(2) Show that $c|0$, for all $c \in \mathbb{Z}, c \neq 0$.

(3) Show that $1|z$ for all $z \in \mathbb{Z}$.

(4) Use the fact that if $a|b$ and $b \neq 0$ then $|a| \leq |b|$ to prove that if $a|b$ and $b|a$ then $|a| = |b|$.

(5) Use the previous two results to prove that if $a|1$ then $|a| = 1$.

(6) Let P(n) be the proposition that $2|(n^2 + n)$. Use Mathematical induction to prove that P(n) is true for all natural numbers n.

(7) Let P(n) be the proposition that $2|(n^2 + n)$. Now use case analysis to show that (n) is true for all natural numbers n.

(8) Let P(n) be the proposition that $4|(5^n - 1)$. Use Mathematical induction to prove that P(n) is true for all natural numbers n.

(9) Let $a, b, c \in \mathbb{Z}$ such that $c|a$ and $c|b$. Let r be the remainder of the division of b by a, that is there is a $q \in \mathbb{Z}$ such that $b = qa + r, 0 \leq r < |b|$. Show that under these condition we have $c|r$.

(10) Consider the function A, such that A(1) = 1, A(2) = 2, A(3) = 3, and for $n \in \mathbb{N}, n \geq 4$, $A(n) = A(n-1) + A(n-2) + A(n-3)$.
  (a) Find values $A(n)$ for $n = 4, 5, 6$.
  (b) Use the second form of mathematical induction to prove that $A(n) \leq 3^n$ for all natural numbers n.

(11) Let a = 1763, and b = 42
  (a) Find $\gcd(a, b)$. Show the steps used by Euclid's algorithm to find $\gcd(a, b)$.

(b) Find integers x, y such that $\gcd(a, b) = ax + by$
(c) Find lcm(a,b)

(12) Prove $\gcd(a, a + k)$ divides k.

(13) If a and b are relatively prime, that is $\gcd(a, b) = 1$ then we can always find integers x, y such that $1 = ax+by$. This fact will be useful to prove the following proposition.

Suppose p is a prime such that $p|ab$, that is p divides the product ab, then $p|a$ or $p|b$.