CISC-102 Winter 2020

Quiz 1

January 29, 2020

Student ID: Solutions

Read the questions carefully. Please clearly state any assumptions that you make that are not explicitly stated in the question.

Please answer all questions in the space provided. Use the back of pages for scratch work. There are 5 pages to this quiz.

SCRAP PAPER OR CALCULATORS ARE NOT PERMITTED.

Special symbols are used as follows:

$\mathbb{N}$ denotes the set of positive integers $\{1, 2, 3, \ldots\}$,

$\mathbb{Z}$ denotes the set of integers $\{\ldots, -2, -1, 0, 1, 2, \ldots\}$,

You may find these definitions useful:

The *relative complement* of a set B with respect to set A is denoted by $A \setminus B$, and is defined as

$$A \setminus B = \{x : x \in A, x \notin B\}.$$

The *symmetric difference* of sets A and B is denoted by $A \oplus B$ and is defined as:

$$A \oplus B = (A \cup B) \setminus (A \cap B).$$

The *complement* of a set A denoted by A^c is defined as:

$$A^c = \{x : x \notin A\}.$$

For True or False questions mark each entry **T** or **F**. For example:

1. Which of the following is true or false?

   (a) 1 is an integer. **T**

   (b) $7 < 3$. F

   (c) $-1 \in \mathbb{N}$. F

Each entry of a True or False question is worth 1 point. For all other questions (x) denotes the question is worth x points.

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1. Let $S = \{a, b, c, d, e, f, g\}$. Let $P(S)$ denote the power set of S, that is the set of all subsets of S. Which of the following is true or false? Mark each entry **T** or **F**.

   (a) $P(S)$ is a partition of S. F
   (b) $|P(S)| = 2^7$. T
   (c) $S \subseteq P(S)$. F
   (d) $S_1 = \{a, a, a\}$ is a subset of S. T
   (e) $S_2 = \{a, g, h\}$ is a superset of S. F
   (f) $S_2 = \{a, g, h\}$ is a subset of S. F

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2. Let A and B be sets such that $A = \{1, 2, 3, 5, 9\}$ and $B = \{2, 4, 6, 8\}$. Which of the following is true or false? Mark each entry **T** or **F**.

   (a) $A \subseteq B$ F
   (b) $B \subseteq A$ F
   (c) $A \cap B = \{2\}$ T
   (d) $|B| \leq |A|$ T

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3. If A is a set of n elements and $a \in A$. Which of the following is true or false? Mark each entry **T** or **F**.

   (a) $|P(A \setminus \{a\})| = 2^n$ F
   (b) $|P(A) \setminus \{\{a\}\}| = 2^{n-1}$ F
   (c) $|P(A) \setminus \{\{a\}\}| = 2^n - 1$ T
   (d) $|P(A \setminus \{a\})| = \frac{1}{2}2^n$ T

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4. The principle of inclusion and exclusion for determining the cardinality (size) of the union of two sets A and B can be expressed as:

$$|A \cup B| = |A| + |B| - |A \cap B|$$

   Suppose $|A| = 10$ and $|B| = 8$. Which of the following is true or false? Mark each entry **T** or **F**.

   (a) if $A \cap B = \emptyset$ then $|A \cup B| < 18$. F
   (b) if $|A \cap B| = 1$ then $|A \cup B| < 18$. T
   (c) if $|A \cup B| = 12$ then $A \cap B = \emptyset$. F
   (d) if $\cancel{A \subseteq B}$ then $|A \cup B| < 18$. T
   $B \subseteq A$

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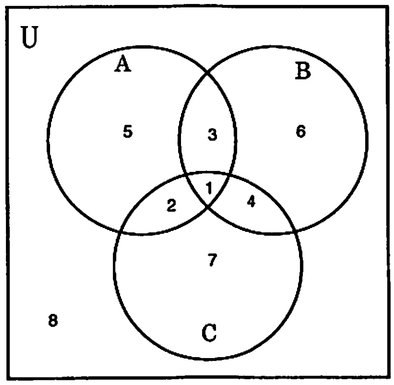

5. Match each of the following set expressions with a region in the Venn diagram shown above by writing the region number next to the expression. I have illustrated what to do with regions 1 and 8. Note some regions may not be represented by an expression and others by more than one expression.

   (a) $U \setminus (A \cup B \cup C)$. 8
   (b) $A \cap B \cap C$. 1
   (c) (1) $A^c \cap B^c \cap C$. 7
   (d) (1) $A \cap B^c \cap C^c$. 5
   (e) (1) $A \cap B^c \cap C$. 2
   (f) (1) $A^c \cap B \cap C$. 4
   (g) (1) $C \setminus (A \cup B)$. 7
   (h) (1) $(A \cap C) \setminus B$. 2
   (i) (1) $(B \setminus A) \cap (B \setminus C)$. 6
   (j) (1) $(A \cup B \cup C)^c$ 8

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6. Let $A = \{1, 2\}, B = \{1, 2, 3, 5\}$.

   (a) (1)What is $A \oplus B$?

   $\{3, 5\}$

   (b) (1) What is $B \oplus A$?

   $\{3, 5\}$

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   (c) (1) Suppose $C \subseteq D = D \subseteq C$. Then what is the value of $C \oplus D$?

   $\emptyset$

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7. Let $A_i = \{i, 2i, 3i, \ldots\}$ for all $i \in \mathbb{N}, i \geq 1$.

   (a) (1) What is A_1?

   $\{1, 2, 3 \ldots\} = \mathbb{N}$

   (b) (1) What is $\bigcup_{i=1}^{n} A_i$?

   $A_1 \cup A_2 \cup \cdots \cup A_n = \mathbb{N}$

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   (c) (1) What is $A_1 \cap A_2$?

   A_2

   (d) (1) What is $(A_1 \cap A_2 \cap A_3 \cap A_4) \cap A_{12}$?

   A_{12}

8. (2) Consider the proposition $P(n)$

$$\sum_{i=1}^{n} 2i = (n-1)(n+2)$$

The following is a flawed proof that claims to show that the proposition $P(n)$ is true for all natural numbers $n \geq 1$.

Induction Hypothesis: P(k) is true for a fixed value $k \in \mathbb{N}, k \geq 1$.

Induction Step: (We show $\sum_{i=1}^{k+1} 2i = (k+1-1)(k+1+2) = k(k+3)$)

$$\begin{aligned}
\sum_{i=1}^{k+1} 2i &= \sum_{i=1}^{k} 2i + 2k + 2 \\
&= (k-1)(k+2) + 2k + 2 \text{by the Ind. Hyp.} \\
&= k^2 + k - 2 + 2k + 2 \\
&= k^2 + 3k = k(k+3)
\end{aligned}$$

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What is wrong with the proof?

The base case is missing.

9. (5) Consider the proposition $P(n)$

$$\sum_{i=1}^{n} 2i = n(n+1)$$

Use mathematical induction to prove that $P(n)$ is true for all natural numbers $n \geq 1$.

Base: $\sum_{i=1}^{1} 2i = 2 = 1(1+1)$

Ind. Hyp.: Assume $\sum_{i=1}^{k} 2i = k(k+1)$ for a fixed $k \in \mathbb{N}$ $k \geq 1$

Ind. Step. $\sum_{i=1}^{k+1} 2i = \sum_{i=1}^{k} 2i + 2k+2$

$= k(k+1) + 2k + 2$ (Ind Hyp.)

$= k^2 + 3k + 2$

$= (k+1)(k+2)$