CISC-102 WINTER 2020

HOMEWORK 1 SOLUTIONS

Problems

(1) Rewrite the following statements using set notation, and then give an example by listing members of sets that match the description. For example: A is a subset of C. Answer: $A \subseteq C$. $A = \{1, 2\}$, $C = \{1, 2, 3\}$.

There are many different solutions to these questions. I have shown several possibilities.

(a) The element 1 is not a member of (the set) A.
$1 \notin$ A. A = {2, 4}.

(b) The element 5 is a member of B.
$5 \in$ B. B = {5,6}

(c) A is not a subset of D.
A $\nsubseteq$ D. A = {2, 4} and D = {42, 18}.

(d) E and F contain the same elements.
E = F. E = F = {7}. E $\subseteq$ F and F $\subseteq$ E.

(e) A is the set of integers larger than three and less than 12.
A = $\{ x : x \in \mathbb{Z}, 3 < x < 12 \}$. A = { 4, 5, 6, 7, 8, 9, 10, 11}.

(f) B is the set of even natural numbers less than 15.
B = $\{ 2x : x \in \mathbb{N}, x < 8\}$. B = {2,4,6,8,10,12,14}.

(g) C is the set of natural numbers x such that $4 + x = 3$.
C = $\{x : x \in \mathbb{N}, 4 + x = 3 \}$. C = $\emptyset$.

(2) $A = \{x : 3x = 6\}$. $A = 2$, true or false? A = {2}. A $\neq$ 2, so the statement is false.

(3) Which of the following sets are equal $\{r, s, t\}$, $\{t, s, r\}$, $\{s, r, t\}$, $\{t, r, s\}$. They are all equal. The order in which elements are written in a set is not important, unless ellipses "... " are used to denote a sequence. For example $x = \{1, 2, \dots, 10\}$.

(4) Consider the sets $\{4, 2\}$, $\{x : x^2 - 6x + 8 = 0\}$, $\{x : x \in \mathbb{N}, x \text{ is even}, 1 < x < 5\}$. Which one of these sets is equal to $\{4, 2\}$?

They are all equal.

(5) Which of the following sets are equal: $\emptyset$, $\{\emptyset\}$, $\{0\}$. None are equal. $\{\emptyset\}$ is a set within a set. 0 is a number not a set, and definitely not the empty set.

(6) Explain the difference between $A \subseteq B$, and $A \subset B$, and give example sets that satisfy the two statements.

$A \subseteq B$ is pronounced as "A is a subset of B" implying that A is a subset of B that may also be equal to A. A = B = {1}. $A \subset B$ is pronounced "A is a proper subset of B " implying that A is strictly a subset of B, and there is at least one element of B that is not an element of A. A = {1} , B ={1,2}.

(7) Consider the following sets $A = \{1, 2, 3, 4\}$, $B = \{2, 3, 4, 5, 6, 7\}$, $C = \{3, 4\}$, $D = \{4, 5, 6\}$, $E = \{3\}$.

(a) Let X be a set such that $X \subseteq A$ and $X \subseteq B$. Which of the sets could be X? For example X could be C, or X could be E. Are there any other sets that could be X?

X could also be {2,3,4}.

(b) Let $X \not\subseteq D$ and $X \not\subseteq B$. Which of the the sets could be X? Set A is the only set from the list that is not a subset of D and not a subset of B. There are infinitely more possibilities of sets that satisfy these requirements. For example all sets $X_i = \{x : x \in \mathbb{N}, x > 8 + i\}$ for all values of $i \in \mathbb{N}$, represents an infinite collection of sets that are not subsets of B or D.

(c) Find the smallest set M that contains all five sets.

M = {1,2,3,4,5,6,7}

(d) Find the largest set N that is a subset of all five sets. $N = \emptyset$

(8) Is an "element of a set", a special case of a "subset of a set"?

No, an element of a set is not a subset.

(9) Phrase the handshake counting problem using set theory notation.

How many two element subsets can be chosen from an n element set?

(10) List all of the subsets of $\{1, 2, 3\}$.

$\emptyset$, {1}, {2}, {3}, {1,2}, {1,3}, {2,3}, {1,2,3}.

(11) Let $A = \{a, b, c, d\}$.

(a) List all the subsets of A containing a.

$\{a\}, \{a, b\}, \{a, c\}, \{a, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{a, b, c, d\}$

(b) List all the subsets of A not containing b

$\emptyset$ $\{a\}, \{c\}, \{d\}, \{a, c\}, \{a, d\}, \{c, d\}, \{a, c, d\}$

(c) Is it a coincidence that the previous two answers have exactly the same number of subsets? Explain.

Not a coincidence. Observe that the total number of subsets of A is exactly 16. Since there are 8 subsets of A with an a then it is easy to conclude that the number of subsets of A without an a is also 8. So it follows that the number of subsets of A without a b is 8.

(d) List all the subsets of A containing both a and b.

$\{a, b\}, \{a, b, c\}, \{a, b, d\}, \{a, b, c, d\}$

(e) List all the subsets of A containing a but not containing b.

$\{a\}, \{a, c\}, \{a, d\}, \{a, c, d\}$

(f) Define an *even subset* of a set, as any subset that has an even number of elements. List all even subsets of A.

$\emptyset$, $\{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, c, d\}$

(g) Define an *odd subset* of a set, as any subset that has an odd number of elements. List all odd subsets of A.

$\{a\}, \{b\}, \{c\}, \{d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}$.

(h) Is it a coincidence that A has the same number of even as odd subsets? Explain.

Not a coincidence. Observe that the number of even subsets of A is half the total number of subsets of A.