CISC-102 WINTER 2020

HOMEWORK 5 SOLUTIONS

(1) Consider the following relations on the set $A = \{1, 2, 3\}$:

- $R = \{(1, 1), (1, 2), (1, 3), (3, 3)\}$,
- $S = \{(1, 1), (1, 2), (2, 1), (2, 2), (3, 3)\}$,
- $T = \{(1, 1), (1, 2), (2, 2), (2, 3)\}$,
- $A \times A$.

For each of these relations determine whether it is symmetric, antisymmetric, reflexive, or transitive.

S and A × A are symmetric.

R and T are antisymmetric.

S and A × A are reflexive.

R, S and A × A are transitive.

(2) Explain why each of the following binary relations on the set $S = \{1, 2, 3\}$ is or is not an equivalence relation on S.

(a) $R_1 = \{(1, 1), (1, 2), (3, 2), (3, 3), (2, 3), (2, 1)\}$
(b) $R_2 = \{(1, 1), (2, 2), (3, 3), (2, 1), (1, 2),$
$(3, 2), (2, 3), (3, 1), (1, 3)\}$
(c) $R_3 = \{(1, 1), (2, 2), (3, 3), (3, 1), (1, 3)\}$

R_1, is neither reflexive nor transitive so it's not an equivalence relation. R_1 is symmetric.

R_2 is reflexive, symmetric, and transitive so it is an equivalence relation.

R_3 is reflexive, symmetric and transitive, so it is an equivalence relation.

(3) Consider a relation W on the set $\mathbb{Z}$ defined as: $W = \{(x, y) \in \mathbb{Z} \times \mathbb{Z} : x^2 = y^2\}$. Show that W is an equivalence relation.

W is reflexive, because $x^2 = x^2$ for all $x \in \mathbb{Z}$. W is symmetric, because if $x^2 = y^2$ then $y^2 = x^2$. W is transitive, because if $x^2 = y^2$ then $y^2 = z^2$ then $x^2 = z^2$.

Let n be an arbitrary integer What are the elements of the equivalence class $[n]$.

$[n] = \{-n, n\}$.

(4) Let $A = \{1, 2, 3\}$ and $B = \{1, 2\}$. We can define a relation on the powerset of A, $P(A)$, as $\mathrm{R} = \{(X, Y) \in P(A) \times P(A) : X \cap B = Y \cap B\}$. Show that R is an equivalence relation. What is the partition of $P(A)$ with respect to R?

R is reflexive because $X \cap B = X \cap B$ for all $X \in P(A)$. R is symmetric because if $X \cap B = Y \cap B$ then $Y \cap B = X \cap B$. R is transitive because if $X \cap B = Y \cap B$ and $Y \cap B = Z \cap B$ then $X \cap B = Z \cap B$.

What is the partition of $P(A)$ with respect to R?

The partition of $P(A)$ with respect to R is $\{[\{1\}, \{1, 3\}], [\{2\}, \{2, 3\}], [\{1, 2\}, \{1, 2, 3\}], [\emptyset, \{3\}]\}$.

(5) Let R be a relation on the set of Natural numbers such that $(a, b) \in \mathrm{R}$ if $a \geq b$. Show that the relation R on $\mathbb{N}$ is a partial order.

R is reflexive because for all $a \in (N)$ $a \geq a$. R is antisymmetric because for all $a, b \in \mathbb{N}, a \neq b$ we have either $a \geq b$ or $b \geq a$ but not both. R is transitive because for all $a, b, c \in \mathbb{N}$, if $a \geq b$ and $b \geq c$, we have $a \geq c$.

(6) Which of the following relations on the set $S = \{1, 2, 3, 4, 5, 6\}$ is a function?

- R = {(1,1), (2,2), (3,2), (4,2), (5,3),(6,3) }
- S = {(1,1), (2,2), (3,2), (4,2), (5,3),(6,3), (1,4) }
- T = {(1,1), (2,2), (3,3), (4,4) }
- $S \times S$

R is a function, because every element in the domain, S, has a distinct image.

S is not a function, because 1 has two different images, due to the pairs (1,1), and (1,4).

T is not a function because the elements of S 5 and 6 do not have images.

$S \times S$ is not a function because every element of S has multiple images.