CISC-102 FALL 2018

HOMEWORK 2

Please work on these problems and have your solutions ready by next Thursday. Assignments will **not** be collected for grading.

READINGS

Read sections 1.5, 1.6, 1.7, 1.8 of *Schaum's Outline of Discrete Mathematics*.

Read sections 1.1, 1.2 and 1.3 again (if you did not understand things last week) of *Discrete Mathematics Elementary and Beyond*.

PROBLEMS

(1) Illustrate DeMorgan's Law $(A \cap B)^c = A^c \cup B^c$ using Venn diagrams.

(2) Let $A_i = \{1, 2, 3, \ldots, i\}$ for all $i \in \mathbb{N}$. For example $A_4 = \{1, 2, 3, 4\}$.

    (a) What are the elements of the set

$$\cup_{i=1}^{n} A_i?$$

    (b) What are the elements of the set

$$\cap_{i=1}^{n} A_i?$$

(3) Observe that $A \subseteq B$ has the same meaning as $A \cap B = A$. Draw a Venn diagram to illustrate this fact.

(4) Use a Venn diagram to show that if $A \subseteq B$ **and** $B \subseteq C$, then $A \subseteq C$.

(5) Use the Principle of Exclusion and Inclusion to show that $|A \cup B| + |A \cap B| = |A| + |B|$. (It may help your understanding if you first explore an example such as A = {1,2,3} and B = {3,4})

(6) What are the cardinalities of the following sets?

    (a) A = {winter, spring, summer, fall}.

    (b) B = $\{x : x \in \mathbb{Z}, 0 < x < 7 \}$.

    (c) $\mathcal{P}(B)$, that is, the power set of B.

    (d) C = $\{ x : x \in \mathbb{N}, x \text{ is even} \}$

(7) Suppose that we have a sample of 100 students at Queen's who take at least one of the following language courses, French-101, Spanish-101, German-101. Also suppose that 65 take French-101, 45 take German-101, 42 take Spanish 101, 20 take French-101 and German-101, 25 take French-101 and Spanish-101, and 15 take German-101 and Spanish-101.

(a) How many students take all three language courses? (HINT: Use the Principle of Inclusion and Exclusion to write an expression representing these students and the classes they take.)

(b) Draw a Venn diagram representing these 100 students and fill in the regions with the correct number.

(c) How many students take exactly 1 of these courses?

(d) How many students take exactly 2 of these courses?

(8) Recall that the union operation is associative, that is $A \cup (B \cup C) = (A \cup B) \cup C$. Show that the relative complement set operation is not associative, that is, $A \backslash (B \backslash C) = (A \backslash B) \backslash C$, is incorrect for some sets A, B, C. (Note if relative complement is associative then the equation must be true for all sets A, B, C.)

(9) Consider a set S of n elements, such that $\{a, b\} \subseteq S$.

(a) What is the cardinality of the power set of $S \setminus \{a\}$?

(b) What is the cardinality of the power set of $S \setminus \{a, b\}$?

(c) How many subsets of S are there that contain the element a?

(d) How many subsets of S are there that contain the element a and exclude the element b?