

## CISC-102 FALL 2018

### HOMEWORK 6

Assignments will **not** be collected for grading.

#### READINGS

Read sections 11.6 and 11.8 of *Schaum's Outline of Discrete Mathematics*.

Read section 6.6 ( Don't worry if the theorems of this section seem daunting. The first 3 pages of the section do give a good explanation of gcd, and lcm.) of *Discrete Mathematics Elementary and Beyond*.

#### PROBLEMS

- (1) Let  $a, b \in \mathbb{R}$ . Prove  $(ab)^n = a^n b^n$ , for all  $n \in \mathbb{N}$ . Hint: Use induction on the exponent  $n$ .
- (2) Let  $a = 1763$ , and  $b = 42$ 
  - (a) Find  $\gcd(a, b)$ . Show the steps used by Euclid's algorithm to find  $\gcd(a, b)$ .
  - (b) Find integers  $x, y$  such that  $\gcd(a, b) = ax + by$
  - (c) Find  $\text{lcm}(a, b)$
- (3) Prove  $\gcd(a, a + k)$  divides  $k$ .
- (4) If  $a$  and  $b$  are relatively prime, that is  $\gcd(a, b) = 1$  then we can always find integers  $x, y$  such that  $1 = ax + by$ . This fact will be useful to prove the following proposition.  
Suppose  $p$  is a prime such that  $p|ab$ , that is  $p$  divides the product  $ab$ , then  $p|a$  or  $p|b$ .
- (5) Find all Natural numbers between 1 and 50 that are congruent to 4 (mod 11).
- (6) Find two Natural numbers  $a$  and  $b$  such that  $2a \equiv 2b \pmod{6}$ ,  
but  $a \not\equiv b \pmod{6}$ .