1. (2)Which set corresponds to the following statement? B is the set of all natural numbers less than 21.

   (a) $B = \{x : x \in \mathbb{Z}, x < 21\}$
   (b) $B = \{\} \cup \{x : x \in \mathbb{N}, x < 21\}$
   (c) $B = \{2x : x \in \mathbb{N}, x < 11\}$
   (d) $B = \{1, 2, 4\} \cup \{2, 4\} \cup \{x : x \in \mathbb{N}, 4 \leq x < 21\}$

2. (2) A group of 33 children are surveyed, all play either hockey, soccer, or basketball. It is found that 15 play hockey and 18 play soccer. Furthermore, 8 play both hockey and soccer ~~(and not basketball)~~, 6 play both soccer and basketball ~~(and not hockey)~~, and 4 play both basketball and hockey ~~(and not soccer)~~. If 3 children play all three sports, how many children play basketball?

   (a) 7
   (b) 10
   (c) 15
   (d) not enough information

3. (2) Let A and B be sets such that $A = \{1, 2, 3, 5, 9\}$ and $B = \{2, 4, 6, 8\}$. Which one of the following statements is true?

   (a) $A \subseteq B$
   (b) $B \subseteq \{2x : x \in \mathbb{N}\}$
   (c) $A = B^c$
   (d) not enough information

4. (2) If $A = A \cup B$ which of the following is true?

   (a) $A \subset B$
   (b) $A^c = A^c \cap B^c$
   (c) $B^c = A^c$
   (d) $A = \emptyset$

5. (2) Which of the following sets is equal to $\{1,3\}$?

   (a) $\{x : x^2 - 2x - 1 = 0\}$
   (b) $\{x : x \text{ is odd}, \ 0 < x < 5\}$
   (c) $\{3,1\} \cap \{2,3\}$
   (d) $\{x : x \in \mathbb{N}, 1 \leq x \leq 3\}$

6. (2) If set $A = \{1,2\}$, then what is $|P(A)|$, the cardinality of the powerset of A, or in plain English the number of subsets which can be formed from A?

   (a) 2
   (b) 3
   (c) 4
   (d) 5

7. (2) Consider the recursive function defined as $Fact(n) = Fact(n-1) \times n$. What is the value of $Fact(4)$?

   (a) 24
   (b) 16
   (c) 4
   (d) not enough information

8. (2)Consider the function $f : \mathbb{Z} \to \mathbb{Z}$ such that $f(x) = 6x$. Which of the following statements is true?

   (a) f is a one-to-one function.
   (b) f is an onto function.
   (c) f is neither one-to-one or onto.
   (d) f is a bijection, that is f is both one-to-one and onto.

9. (2) Which of the following sets is a proper subset of $A = \{1,2,3,5,6\}$?

   (a) $\{3,4,5,6\}$
   (b) $\{\emptyset\}$
   (c) $\{\}$
   (d) $\{2,1,5,3,6\}$

10. Consider the proposition $P(n)$

$$\sum_{i=4}^{n} i = \frac{(n-3)(n+4)}{2}.$$

Answer the following questions to prove that $P(n)$ is true for all $n \in \mathbb{N}, n \geq 4$.

(a) (2) What is the base case?

For $n=4$ $\displaystyle\sum_{i=4}^{4} i = 4 = \frac{(4-3)(4+4)}{2}$

(b) (2) What is the induction hypothesis?

Assume that $P(k)$ is true for a fixed $k \in \mathbb{N}$ $k \geq 4$.

That is. $\displaystyle\sum_{i=4}^{k} i = \frac{(k-3)(k+4)}{2}$

(c) (3) What is the induction step?

$$\sum_{i=4}^{k+1} i = \sum_{i=4}^{k} i + k+1$$

$$= \frac{(k-3)(k+4)+2k+2}{2}$$

$$= \frac{k^2+3k-10}{2}$$

$$= \frac{(k-2)(k+5)}{2} = \frac{(k+1-3)(k+1+1)}{2}$$

11. Consider a recursive function defined as:

$T(1) = 0$

$T(n) = T(n-1) + (n-1)$ for $n \in \mathbb{N}, n \geq 2$.

(a) (3) What are the values for $T(2), T(3)$, and $T(4)$?

$T(2) = 0+1 = 1 \quad T(3) = 1+2 = 3 \quad T(4) = 3+3 = 6$

(b) (7) Let $P(n)$ be the proposition $T(n) = \frac{n(n-1)}{2}$. Use mathematical induction to prove that $P(n)$ is true for all $n \in \mathbb{N}$.

Base $T(1) = 0 = \frac{1(1-1)}{2}$

I.H. Assume $P(k)$ is true for some fixed $k \in \mathbb{N}$, $k \geq 1$.

I.S. $T(k+1) = T(k) + k$

$= \frac{k(k-1)+2k}{2}$

$= \frac{k^2+k}{2}$

$= \frac{(k+1)(k+1-1)}{2}$