1. (2) Suppose there are 28 different colours a car can be painted. What is the minimum number of cars you would need to see to be sure you've seen two cars with the same colour?

   (a) 28
   (b) 29
   (c) 2
   (d) 56

2. (2) Suppose an outfit consists of a shirt, a pair of pants, a pair of socks, and a pair of shoes. If you have 108 possible outfits and you have 3 shirts, 3 pairs of pants, and 3 pairs of socks , how many pairs of shoes do you have?

   (a) 2
   (b) 3
   (c) 4
   (d) 6

3. (2) Which of the following terms belongs to the expansion of $(x+y)^7$?

   (a) $\binom{7}{1}x^7$
   (b) $\binom{7}{4}x^3y^4$
   (c) $\binom{7}{6}x^6y^2$
   (d) None of the above

4. (2) Consider the sum:

$$S_n = \sum_{i=0}^{n} \binom{n}{i} 3^{n-i}(2)^i.$$

   Which of one of the following is true?

   (a) $S_n = 5^n$
   (b) $S_n = 0$
   (c) $S_n = 6^n$
   (d) None of the above.

5. (2) In how many different ways can the letters BANAN be rearranged?

   (a) $\frac{5!}{3!2!}$
   (b) $\frac{6!}{3!2!}$
   (c) $\frac{5!}{2!2!}$
   (d) None of the above.

6. (2) Consider a bag containing 10 balls numbered from 1 to 10. In how many ways can 5 balls be selected, without ordering, and without replacement, so that all 5 numbers are even or all 5 are odd?

   (a) $\binom{10}{5}\binom{10}{5}$
   (b) $\binom{10}{5} + \binom{10}{5}$
   (c) $\binom{5}{5}\binom{5}{5}$
   (d) $\binom{5}{5} + \binom{5}{5}$

7. (2) Which one of the following equations is true for all natural numbers k.

   (a) $\binom{k+1}{k} = \binom{k+2}{k+1}$
   (b) $\binom{k+1}{2} = \binom{k+2}{k-1}$
   (c) $\binom{k+1}{2} = \binom{k+1}{k-1}$
   (d) $\binom{k+1}{k} = \binom{k+1}{k-1}$

8. (4) Complete the truth table below, adding columns as needed, for the proposition:

$$p \wedge (p \vee q).$$

p	q	p ∨ q	p ∧ (p ∨ q)	
T	T	T	T	
T	F	T	T	
F	T	T	F	
F	F	F	F	

9. Consider the logical argument:

$$(\neg p \rightarrow \neg q) \vdash (q \rightarrow p).$$

(a) (2) Rewrite the logical argument as a logical expression.

$(\neg p \to \neg q) \to (q \to p)$

(b) (4) Complete the truth table below, adding columns as needed to determine whether the argument above is valid or not. After you have completed the table explain your conclusion in a sentence or two.

p	q	$\neg p$	$\neg q$	$\neg p \to \neg q$	$q \to p$	$(\neg p \to \neg q) \to (q \to p)$
T	T	F	F	T	T	T
T	F	F	T	T	T	T
F	T	T	F	F	F	T
F	F	T	T	T	T	T

The logical expression is a tautology. therefore we conclude that the argument is valid.

10. Answer the following questions to prove that:

$$\sum_{m=1}^{n} \binom{m}{m-1} = \binom{n+1}{2}$$

is true for all natural numbers n.

(a) (2) The proof is by induction with the base case n = 1.
Show, by expanding the binomial coefficients that the base case:

$$\sum_{m=1}^{1} \binom{m}{m-1} = \binom{(1+1)}{2} \text{ is true.}$$

$$\frac{1!}{(1-1)!\,1!} = 1 \quad \text{and} \quad \frac{2!}{2!\,0!} = 1$$

(b) (4) For the induction hypothesis we assume that:

$$\sum_{m=1}^{k} \binom{m}{m-1} = \binom{k+1}{2}$$

is true for $k \in \mathbb{N}, k \geq 1$. Now complete the proof with the induction step.

$$\sum_{m=1}^{k+1} \binom{m}{m-1} = \sum_{m=1}^{k} \binom{m}{m-1} + \binom{k+1}{k}$$

$$= \binom{k+1}{2} + \binom{k+1}{k}$$

$$= \frac{(k+1)!}{2!\,(k-1)!} + \frac{(k+1)!}{k!\,1!}$$

$$= \frac{k\,(k+1)!}{2!\,k!} + \frac{2\,(k+1)!}{k!\,2!}$$

$$= \frac{(k+2)(k+1)!}{k!\,2!} = \binom{k+2}{2}$$