CISC-102 FALL 2018

HOMEWORK 1 SOLUTIONS

Problems

(1) Rewrite the following statements using set notation, and then give an example by listing members of sets that match the description. For example: A is a subset of C. Answer: $A \subseteq C$. $A = \{1, 2\}$, $C = \{1, 2, 3\}$.

There are many different solutions to these questions. I have shown several possibilities.

(a) The element 1 is not a member of (the set) A.
$1 \notin$ A. A = $\{2, 4\}$.

(b) The element 5 is a member of B.
$5 \in$ B. B = $\{5,6\}$

(c) A is not a subset of D.
A $\nsubseteq$ D. A = $\{2, 4\}$ and D = $\{42, 18\}$.

(d) E and F contain the same elements.
E = F. E = F = $\{7\}$. E $\subseteq$ F and F $\subseteq$ E.

(e) A is the set of integers larger than three and less than 12.
A = $\{\ x : x \in \mathbb{Z}, 3 < x < 12\ \}$. A = $\{$ 4, 5, 6, 7, 8, 9, 10, 11$\}$.

(f) B is the set of even natural numbers less than 15.
B = $\{\ 2x : x \in \mathbb{N}, x < 8\}$. B = $\{2,4,6,8,10,12,14\}$.

(g) C is the set of natural numbers x such that $4 + x = 3$.
C = $\{x : x \in \mathbb{N}, 4 + x = 3\ \}$. C = $\emptyset$.

(2) $A = \{x : 3x = 6\}$. $A = 2$, true or false? A = $\{2\}$. A $\neq 2$, so the statement is false.

(3) Which of the following sets are equal $\{r, s, t\}$, $\{t, s, r\}$, $\{s, r, t\}$, $\{t, r, s\}$. They are all equal. The order in which elements are written in a set is not important, unless ellipses "… " are used to denote a sequence. For example $x = \{1, 2, \ldots, 10\}$.

(4) Consider the sets $\{4, 2\}$, $\{x : x^2 - 6x + 8 = 0\}$, $\{x : x \in \mathbb{N}, x \text{ is even}, 1 < x < 5\}$. Which one of these sets is equal to $\{4, 2\}$?

They are all equal.

(5) Which of the following sets are equal: $\emptyset$, $\{\emptyset\}$, $\{0\}$. None are equal. $\{\emptyset\}$ is a set within a set. 0 is a number not a set, and definitely not the empty set.

(6) Explain the difference between $A \subseteq B$, and $A \subset B$, and give example sets that satisfy the two statements.

$A \subseteq B$ is pronounced as "A is a subset of B" implying that A is a subset of B that may also be equal to A. A = B = $\{1\}$. $A \subset B$ is pronounced "A is a proper subset of B " implying that A is strictly a subset of B. A = $\{1\}$, B =$\{1,2\}$.

(7) Consider the following sets $A = \{1, 2, 3, 4\}$, $B = \{2, 3, 4, 5, 6, 7\}$, $C = \{3, 4\}$, $D = \{4, 5, 6\}$, $E = \{3\}$.

(a) Let X be a set such that $X \subseteq A$ and $X \subseteq B$. Which of the sets could be X? For example X could be C, or X could be E. Are there any other sets that could be X?

X could also be $\{2,3,4\}$.

(b) Let $X \not\subseteq D$ and $X \not\subseteq B$. Which of the the sets could be X? Set A is the only set from the list that is not a subset of D and not a subset of B. There are infinitely more possibilities of sets that satisfy these requirements. For example all sets $X_i = \{x : x \in \mathbb{N}, x > 8 + i\}$ for all values of $i \in \mathbb{N}$, represents an infinite collection of sets that are not subsets of B or D.

(c) Find the smallest set M that contains all five sets.

M = $\{1,2,3,4,5,6,7\}$

(d) Find the largest set N that is a subset of all five sets. $N = \emptyset$

(8) Is an "element of a set", a special case of a "subset of a set"?

No, an element of a set is not a subset.

(9) Phrase the handshake counting problem using set theory notation.

How many two element subsets can be chosen from an n element set?

(10) List all of the subsets of $\{1, 2, 3\}$.

$\emptyset$, $\{1\}$, $\{2\}$, $\{3\}$, $\{1,2\}$, $\{1,3\}$, $\{2,3\}$, $\{1,2,3\}$.

(11) Let $A = \{a, b, c, d, e\}$. List all the subsets of A containing a but not containing b.

{a}, {a,c}, {a,d}, {a,e}, {a,c,d}, {a,c,e}, {a,d,e}, {a,c,d,e}