CISC-102

HOMEWORK 3

Readings

Read sections 1.8 of *Schaum's Outline of Discrete Mathematics*.
Read section 2.1 of *Discrete Mathematics Elementary and Beyond*.

Problems

(1) Mathematical induction can be used to prove that the sum of the first n natural numbers is equal to $\frac{n(n+1)}{2}$. This can also be stated as:

We can prove that the proposition P(n),

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

is true for all $n \in \mathbb{N}$, by using mathematical induction.

I wrote out the proof, but somehow it got all scrambled as shown below. Rearrange the lines to get the correct proof.

1. **Induction step:** The goal is to show that P$(k+1)$ is true.

2. **Base:** for n =1, $1 = \frac{1(1+1)}{2}$

3. $= \frac{(k+1)(k+2)}{2}$

4. $\sum_{i=1}^{k+1} i = \sum_{i=1}^{k} i + (k+1)$

5. $= \frac{k^2+k+2k+2}{2}$

6. **Induction hypothesis:** Assume that P(k), for Natural numbers $k \geq 1$ is true, that is:

7. $= \frac{k^2+3k+2}{2}$

8. $\sum_{i=1}^{k} i = \frac{k(k+1)}{2}$

9. $= \frac{k(k+1)}{2} + (k+1)$

(2) Prove using mathematical induction that the proposition $\mathrm{P}(n)$,

$$\sum_{i=2}^{n} i = \frac{(n-1)(n+2)}{2}$$

is true for all $n \in \mathbb{N}, n \geq 2$.

(3) Prove using mathematical induction that the proposition $\mathrm{P}(n)$,

$$\sum_{i=3}^{n} i = \frac{(n-2)(n+3)}{2}$$

is true for all $n \in \mathbb{N}, n \geq 3$.

(4) Prove using mathematical induction that the proposition $\mathrm{P}(n)$

$$n! \leq n^n$$

is true for all $n \in \mathbb{N}$.

(5) Given a set of n points on a two dimensional plane, such that no three points are on the same line, it is always possible to connect every pair of points with a line segment. The figure illustrates this showing 5 points, that are pairwise connected with 10 line segments. Prove using mathematical induction that the total number of line segments is $\frac{n(n-1)}{2}$ for any number of points $n \in \mathbb{N}, n \geq 2$.

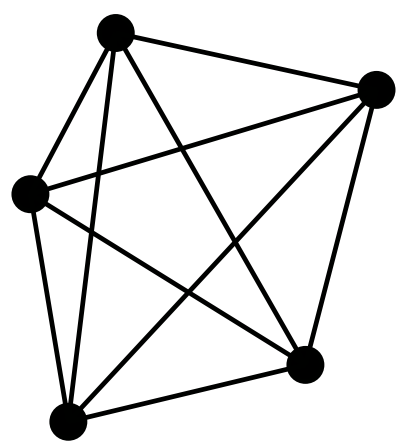

Figure 1. Five points, pairwise connected with 10 line segments.

(6) Consider the following proof that $n + 1 = n$, for all natural numbers n.
Induction Hypothesis: Assume that $k + 1 = k$ for a fixed natural number k.
Induction step:

$$\begin{aligned} k + 2 =& k + 1 + 1 \\ =& k + 1 \text{(apply induction hypothesis)} \\ =& k + 1 \end{aligned}$$

We have shown that $\mathrm{P}(k)$ true implies that $\mathrm{P}(k + 1)$ is true so by the principle of mathematical induction we conclude that $\mathrm{P}(n)$ is true for all $n \in \mathbb{N}$. $\square$

This can't possibly be right! What's wrong?