CISC-102 FALL 2019

HOMEWORK 9

Readings

Read sections 5.3 and chapter 4 of *Schaum's Outline of Discrete Mathematics*.
Read sections 3.1, 3.5, 3.6 of *Discrete Mathematics Elementary and Beyond*.

Problems

(1) Consider the equation

$$\sum_{i=0}^{2} \binom{3}{i}\binom{2}{2-i} = \binom{5}{2}. \tag{1}$$

(a) Use algebraic manipulation to prove that the left hand and right hand sides of equation (1) are in fact equal.
(b) Use a counting argument to prove that the left hand and right hand sides of equation (1) are in fact equal.

(2) Now consider a generalization of the previous equation.

$$\sum_{i=0}^{k} \binom{m}{i}\binom{n}{k-i} = \binom{m+n}{k}. \tag{2}$$

Use a counting argument to prove that the left hand and right hand sides of equation (2) are in fact equal.

(3) In the notes for Week 9 you will find Pascal's triangle worked out for rows 0 to 8. The numbers in row 8 are 1 8 28 56 70 56 28 8 1. Work out the values of rows 9 and 10 of Pascal's triangle with the help of the equation:

$$\binom{n-1}{k} + \binom{n-1}{k-1} = \binom{n}{k}. \tag{3}$$

(4) Show that $\binom{n}{0} = \binom{n-1}{0}$, and that $\binom{n-1}{n-1} = \binom{n}{n}$ by an algebraic argument as well as a counting argument.

(5) Prove that

$$\binom{n}{0} - \binom{n}{1} + \binom{n}{2} - \binom{n}{3} + \cdots + (-1)^n\binom{n}{n} = 0$$

Note that this equation can also be written as follows:

$$\sum_{i=0}^{n} \binom{n}{i} (-1^i) = 0$$

HINT: This can be viewed as a special case of the binomial theorem.

(6) Prove (using mathematical induction on n) that:

$$\sum_{m=0}^{n-1} \binom{m+1}{m} = \binom{n+1}{n-1}$$

is true for all $n \in \mathbb{N}$.

(7) Use a truth table to verify that the proposition $p \vee \neg(p \wedge q)$ is a tautology, that is, the expression is true for all values of p and q.

(8) Use a truth table to verify that the proposition $(p \wedge q) \wedge \neg(p \vee q)$ is a contradiction, that is, the expression is false for all values of p and q.

(9) Use a truth table to show that $p \vee q \equiv \neg(\neg p \wedge \neg q)$.

(10) Show that the following argument is valid.

$$p \to q, \neg q \vdash \neg p$$

(11) Let A = {1,2,3,4,5}. Determine the truth value of each of the following statements.
  (a) $(\exists x \in A)(x + 2 = 7)$
  (b) $(\forall x \in A)(x + 2 < 8)$
  (c) $(\exists x \in A)(x + 3 < 2)$
  (d) $(\forall x \in A)(x + 3 \leq 9)$