

CISC271
Fall 2006
Homework for week 7
in preparation for quiz 3
Solutions

This homework is about piece-wise interpolation.

Recktenwald Chapter 10. questions 29, 31, 32

For question 32 use the Matlab spline with the not-a-knot (default) end condition as well as the so called “fixed slope” end condition, use $y' = 0$ at the ends. Use `help spline` to see how to do this.

10-29 Recall we have

$$P_i(x) = a_i + b_i(x - x_i) + c_i(x - x_i)^2 + d_i(x - x_i)^3$$

$$P'_i(x) = b_i + 2c_i(x - x_i) + 3d_i(x - x_i)^2$$

Let $h_i = x_i - x_{i-1}$. We will use the equations (10.39) (page 539 of Recktenwald) for a_i, b_i, c_i , and d_i .

$$a_{i-1} = y_{i-1} \tag{1}$$

$$b_{i-1} = b_{i-1} \tag{2}$$

$$c_{i-1} = \frac{3f[x_{i-1}, x_i] - 2b_{i-1} - b_i}{h_i} \tag{3}$$

$$d_{i-1} = \frac{b_{i-1} - 2f[x_{i-1}, x_i] + b_i}{h_i^2} \tag{4}$$

Thus $P_i(x_i) = y_i = a_i$ and $P'_i(x_i) = b_i$. It remains to show that $P_{i-1}(x_i) = a_i$ and that $P'_{i-1}(x_i) = b_i$.

Thus,

$$\begin{aligned}
 P_{i-1}(x_i) &= y_{i-1} + b_{i-1}h_i + c_{i-1}h_i^2 + d_{i-1}h_i^3 \\
 &= y_{i-1} + b_{i-1}h_i + h_i(3f[x_{i-1}, x_i] - 2b_{i-1} - b_i) + h_i(b_{i-1} - 2f[x_{i-1}, x_i] + b_i) \\
 &= y_{i-1} + h_i f[x_{i-1}, x_i] \\
 &= y_{i-1} + h_i(y_i - y_{i-1})/h_i \\
 &= y_i
 \end{aligned}$$

And,

$$\begin{aligned}
 P'_{i-1}(x_i) &= b_{i-1} + 2c_{i-1}h_i + 3d_{i-1}h_i^2 \\
 &= b_{i-1} + 2(3f[x_{i-1}, x_i] - 2b_{i-1} - b_i) + 3(b_{i-1} - 2f[x_{i-1}, x_i] + b_i) \\
 &= b_i
 \end{aligned}$$

10-31 No it isn't, because one needs to evaluate the entire system of linear equations to determine any of the coefficients.

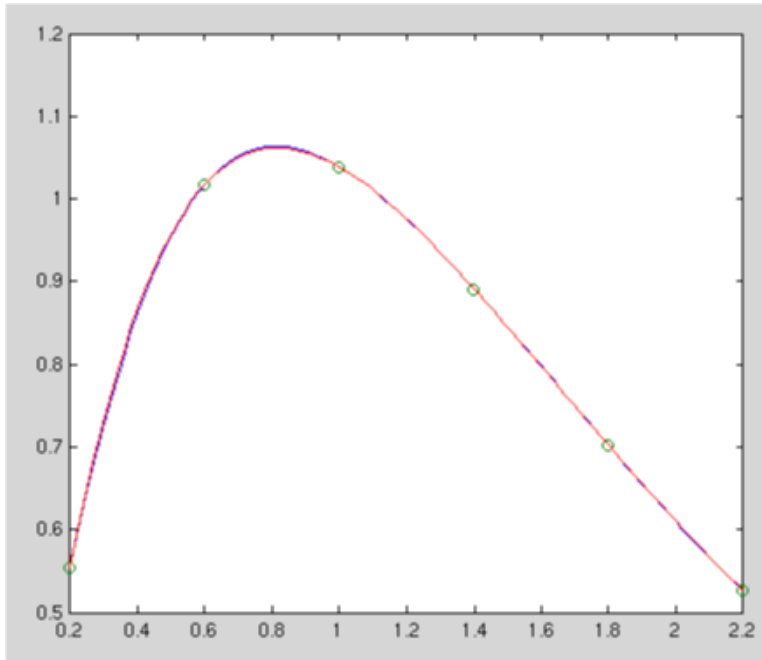
10-32 Here are the Matlab instructions that I used.

```

> x = [ 0.2000    0.6000    1.0000    1.4000    1.8000    2.2000 ];
> y = [ 0.5535    1.0173    1.0389    0.8911    0.7020    0.5257 ];
> xx = linspace(0.2,2.2);
> yy = spline(x,y,xx);
> plot(xx,yy,x,y,'o',xx, sqrt(12.5) .* xx .* exp(-sqrt(1.5).*xx))

```

And here is the plot that I obtained.



For the “fixed slope” slope = 0 end conditions I used

```
yy = spline(x,[0 y 0],xx);
```

The plot did not look as nice. Here it is:

