

## Terminology

### Grouping Terms

$(\cdot)$	parentheses are used to group scalar terms
$[\cdot]$	brackets are used to group vectors, 1-forms, and matrices

### Scalars and Vectors

$a, b, c, d, r$	real number, possibly indexed; typically, known values
$h$	real number <u>or</u> function, <i>context dependent</i> ; e.g., $h(x)$ or $h_{ij}$
$i, j, k, q$	index; positive integer
$l$	<i>context dependent</i> positive integer as an index <u>or</u> real number as a level of a function
$m$	size of an objective vector, i.e., vector is in $\mathbb{R}^m$
$n$	size of a search vector, i.e., vector is in $\mathbb{R}^n$
$p$	<i>context dependent</i> ; probability <u>or</u> positive integer <u>or</u> scalar-valued function, possibly indexed
$w_0, x_0, y_0$	real number; typically, a specific value
$x_i, y_i, z_i$	real number, <i>context dependent</i> ; entry of a vector <u>or</u> $i^{\text{th}}$ iterant of a sequence of scalars
$\vec{a}, \vec{b}, \vec{c}, \vec{d}, \vec{r}$	real vector, possibly indexed; typically, known values
$\vec{e}_i$	elementary vector; $j^{\text{th}}$ entry is 0 if $j \neq i$ and is 1 if $j = i$
$f, g$	scalar-valued function, possibly indexed; e.g., $f : \mathbb{R} \rightarrow \mathbb{R}$ or $f : \mathbb{R}^n \rightarrow \mathbb{R}$
$\vec{f}, \vec{g}, \vec{p}$	vector-valued function, e.g., $\vec{f} : \mathbb{R}^n \rightarrow \mathbb{R}^m$
$H(\underline{P})$	entropy of probability distribution $\underline{P}$
$\mathcal{L}(\vec{x}, \vec{\lambda}, \vec{\mu})$	Lagrange function of vector $\vec{x}$ and multiplier(s) $\vec{\lambda}$ and $\vec{\mu}$
$\mathcal{L}_D(\vec{\lambda}, \vec{\mu})$	Lagrange dual function of multiplier(s) $\vec{\lambda}$ and $\vec{\mu}$
$\vec{q}, \vec{u}, \vec{w}, \vec{x}, \vec{y}, \vec{z}$	real vector, possibly indexed
$\vec{v}$	real vector, possibly indexed, <i>context dependent</i> ; general vector <u>or</u> eigenvector of a matrix
$\vec{w}_0, \vec{x}_0, \vec{y}_0$	real vector; typically, a specific value
$\vec{w}_j, \vec{x}_j, \vec{y}_j, \vec{z}_j$	real vector; typically, $j^{\text{th}}$ iterant of a sequence or $j^{\text{th}}$ instance of data
${}_i\vec{w}, {}_i\vec{x}$	real vector; typically, weight vector or data vector for the $i^{\text{th}}$ layer of an artificial neural network
${}^i\vec{w}$	real vector; for an artificial neural network with $p$ layers, ${}^i\vec{w} \stackrel{\text{def}}{=} [{}_i\vec{w} \dots {}_p\vec{w}]^T$

## 1-Forms

$\underline{a}, \underline{u}, \underline{v}, \underline{x}$

real 1-form in  $\mathbb{R}^n$  with  $i^{\text{th}}$  entry  $a_i$ , etc.; computed as a  $1 \times n$  row matrix  $\underline{a} = [a_1 \ a_2 \ \cdots \ a_n]$

$\underline{0}, \underline{1}$

real 1-forms with constant entries

$\underline{z}(M)$

“mean” 1-form of a matrix, where entry  $z_j$  is the mean of column  $j$  of  $M$

$\underline{P}, \underline{Q}$

Probability distributions

## General Matrices

$A, C, M, X, Y, Z$

matrix; entries are  $a_{ij}$ , etc.

$B$

symmetric positive definite matrix

$D, K$

symmetric matrix, positive definite or positive semidefinite

$R$

upper-triangular matrix

$Q, U$

orthogonal matrix

$V$

orthogonal matrix; may give eigenvectors of another matrix

$W$

*context dependent*; general matrix or symmetric positive semidefinite matrix

## Special Functions

$H(u)$

Heaviside step function;  $H(v) \stackrel{\text{def}}{=} \begin{cases} 0 & \text{if } v < 0 \\ 1 & \text{if } v \geq 0 \end{cases}$

## Greek Symbols

$\alpha, \alpha_i, \vec{\alpha}$

real number, *context dependent*; typically a known value or, in SVM, Lagrange multiplier

$\beta, \gamma, \rho$

real numbers, especially as known values

$\eta$

positive real number; typically,  $0 < \eta < 1$  as a learning rate

$\lambda, \lambda_i$

real number, *context dependent*; eigenvalue or Lagrange multiplier or regularizer

$\mu, \mu_i$

real number, *context dependent*; free parameter or Lagrange multiplier or mean value

$\nu$

real number; generally, variable parameter

$\phi, \theta, \psi$

real number

$\vec{\rho}$

real vector, possibly indexed

$\Lambda$

diagonal matrix where  $\lambda_i$  is an eigenvalue of another matrix

$\Sigma$

diagonal matrix, or diagonal-like matrix such as the singular-value matrix

## Differentials

$f', \frac{df}{dt}$	derivative of $f: \mathbb{R} \rightarrow \mathbb{R}$
$f'', \frac{d^2f}{dt^2}$	derivative of $f': \mathbb{R} \rightarrow \mathbb{R}$
$\frac{\partial f}{\partial w_i}(\vec{w})$	partial derivative of $f: \mathbb{R}^n \rightarrow \mathbb{R}$ with respect to $w_i$
$\underline{\nabla} f(\vec{w})$	1-form derivative of $f: \mathbb{R}^n \rightarrow \mathbb{R}$ ; $i^{\text{th}}$ entry is $\frac{\partial f}{\partial w_i}$
$\frac{\partial^2 f}{\partial w_i \partial w_j}(\vec{w})$	partial derivative of $\frac{\partial f}{\partial w_j}: \mathbb{R}^n \rightarrow \mathbb{R}$ with respect to $w_i$
$D_{\vec{v}} f(\vec{w})$	directional derivative of $f: \mathbb{R}^n \rightarrow \mathbb{R}$ at $\vec{w}$ in direction $\vec{v}$
$J_{\vec{f}}(\vec{w}_0)$	Jacobian matrix of $\vec{f}: \mathbb{R}^n \rightarrow \mathbb{R}^m$ , evaluated at $\vec{w}_0$ ; entry $(i, j)$ is $\frac{\partial f_i}{\partial w_j}(\vec{w}_0)$ so the $i^{\text{th}}$ row of $J_{\vec{f}}(\vec{w}_0)$ is $\underline{\nabla} f_i(\vec{w}_0)$
$\nabla^2 f(\vec{w}_0)$	Hessian matrix of $f: \mathbb{R}^n \rightarrow \mathbb{R}$ , evaluated at $\vec{w}_0$ ; entry $(i, j)$ is $\frac{\partial^2 f}{\partial w_i \partial w_j}(\vec{w}_0)$

## Sets

$\mathbb{N}$	set of natural numbers, including zero
$\mathbb{N}_+$	set of positive natural numbers
$\mathbb{R}$	set of real numbers
$\mathbb{R}^n$	set of real vectors, each vector having $n$ entries
$\mathbb{R}_+^n$	set of real vectors where each entry of each vector is non-negative
$\mathbb{R}_{++}^n$	set of real vectors where each entry of each vector is positive
$\mathbb{S}_C(f, l)$	level curve of function $f$ at level $l$
$\mathbb{S}_L(f, l)$	level set of function $f$ at level $l$
$\mathbb{V}$	vector space over the field of real numbers
$\mathbb{A}$	events $a_i$
$\mathbb{A}(M, \vec{c})$	Affine space; set of vectors $\vec{u}$ that are solutions to $M\vec{w} = \vec{c}$
$\mathbb{F}$	Feasible solutions of a constrained optimization problem