Energy aware techniques for certain problems in Wireless Sensor Networks

by

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Abstract

Recent years have witnessed a tremendous amount of research in the field of wireless sensor networks (WSNs) due to their numerous real-world applications in environmental and habitat monitoring, fire detection, object tracking, traffic controlling, industrial and machine-health control and monitoring, enemy-intrusion in military battlefields, and so on. However, reducing energy consumption of individual sensors in such networks and obtaining the expected standard of quality in the solutions provided by them is a major challenge. In this thesis, we investigate several problems in WSNs, particularly in the areas of broadcasting, routing, target monitoring, self-protecting networks, and topology control with an emphasis on minimizing and balancing energy consumption among the sensors in such networks. Several interesting theoretical results and bounds have been obtained for these problems which are further corroborated by extensive simulations of most of the algorithms. These empirical results lead us to believe that the algorithms may be applied in real-world situations where we can achieve a guarantee in the quality of solutions with a certain degree of balanced energy consumption among the sensors.

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Lastly, I thank my wife Mithila whose understanding, inspiration, and constant support provided me with great optimism even in difficult situations while she took

full care of our two beautiful sons Ayaan and Adib.

Statement of Originality

I hereby certify that this PhD thesis is original and proper references are made wherever I use ideas contributed by others.

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V	a set of n nodes representing n points (sensors) in general position in the plane
E	a set of edges representing links among the sensors
v_iv_j	an edge connecting nodes v_i and v_j
WSN	Wireless Sensor Network
UDG	Unit Disk Graph
G	G is a UDG with node set V and edge set E
GG	Gabriel Graph
RNG	Relative Neighborhood Graph
MIS	Maximal Indpendent Set
CDS	Connected Dominating Set
MCDS	Minimum Connected Dominating Set
MST	Minimum Spanning Tree
DP	Domatic Partition
CH	Cluster Head
$CH(X)$	Convex hull of the set of X points in the plane
$N_k(u)$	u's k-hop neighbor set excluding u
$N_k[u]$	u's k-hop neighbor set including u
D_i	Dominating Set at iteration i

Chapter 1

Introduction

Wireless sensor networks (WSNs) offer a wealth of capabilities in interacting with the physical world by collecting, transmitting, and processing data from the environment and responding accordingly. Typical applications of WSNs include environmental monitoring (such as fires, floods, earthquakes, etc.), infrastructure protection, intelligent homes, military battlefield surveillance, rescue operations, observation of chemical and biological processes, and so on.

A WSN consists of a number of small battery-powered sensors, each equipped with limited on-board processing, bandwidth, sensing, transmission and receiving devices. Through the transmitting device, a sensor can transmit a signal (typically radio signals) to a finite distance (known as the *transmission radius*) and all sensors within this distance can receive the signal by their receiving antennae. Since a radio signal is omnidirectional, one can assume the transmission area of a signal is a circular disk centered at the sensor originating the signal, whose radius is equal to the transmission radius.

As there is no physical connection between any two sensors in a WSN, a virtual

link between them is established if they are located within each other's disk, i.e., they are capable of sending and receiving their signals directly. Furthermore, with its sensing device, a sensor can sense certain environmental phenomena such as temperature, light, pressure, humidity, and the presence of some physical objects (e.g., animals or vehicles and so on) within a certain range known as the *sensing radius*. Typically, a sensors's sensing radius is equal to its transmission radius. Thus, after the deployment of sensors, an adhoc (infrasturcture-less) WSN is formed which then begins to operate independently without any external control. However, the underlying physical topology of such networks is dependent on the distribution of the sensors and their transmission radii.

In general, a WSN is used to allow a collection of unattended sensors deployed at arbitrary locations to form a virtual network and provide cooperatively and collectively sensed data about some events of interest to the base station (also called the *sink*). The sink eventually processes the gathered data transmitted by the network to obtain an overall picture of the environment and takes decisions accordingly. Figure 1.1 shows a typical WSN.

1.1 New Challenges

Because of their flexibility, efficacy, low-cost, and ease of deployment, WSNs are gaining attention and expanding their domain of applications. Unlike traditional and electrically-powered wired computer networks, a WSN comes with a unique set of resource constraints such as finite on-board battery power, limited processing ability, and constrained communication bandwidth. Moreover, a typical WSN is expected to work without human intervention for a long time period. These features present

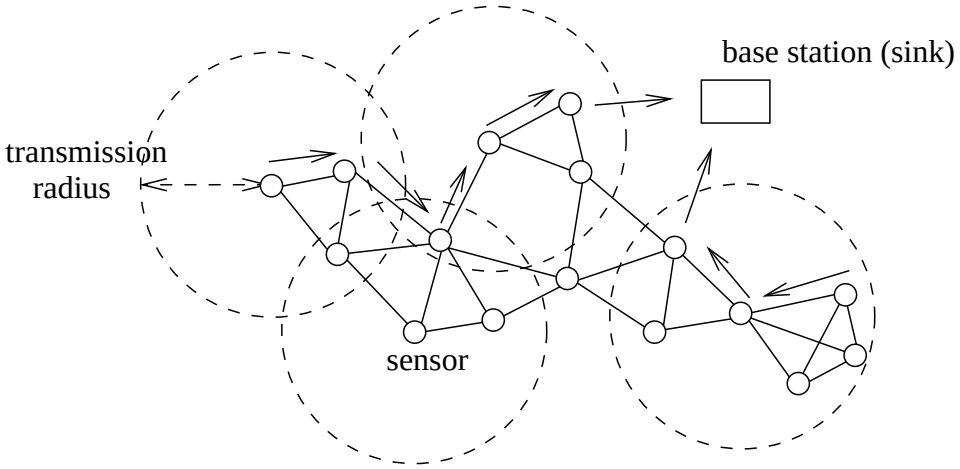


Figure 1.1: A typical wireless sensor network.

unique challenges to come up with new paradigms that can harness the benefits of numerous applications of WSNs.

As sensors are battery-powered, consideration of energy efficiency is of paramount importance in WSNs. Due to the energy constraints of sensors, communication protocols and hardware architectures for WSNs necessitate an energy-aware design to ensure the longevity of the network. Algorithms are expected to address the power constraint of WSNs without compromising the standard of the solution provided by them.

As each transmission of data over a link in WSNs consumes a certain amount of energy (energy consumption is directly proportional to at least the square of the Euclidean distance of the link), special care must be taken so that protocols minimize the amount of communication. Local collaboration among sensors, redundant data suppression, data compression, and avoidance of direct transmission to long-distant sensors are some of the important factors that influence algorithm designers to devise

novel distributed, scalable and energy-efficient solutions for WSNs. Moreover, the dynamic nature of WSNs, e.g., sensor and link failures (due to physical damage or power failure of sensors) requires algorithms to be robust and resilient in such situations.

1.2 Problems and Energy Issues in Wireless Sensor Networks

As mentioned before, reducing energy consumption on individual sensors (or just *nodes*, henceforth we use sensors and nodes interchangeably) in the network and obtaining the expected standard of quality in the solutions provided by such networks is a major challenge. In this thesis, we address a number of problems arising from various applications of WSNs with a view to finding efficient polynomial-time solutions while emphasizing the energy minimization issue of such networks. The problems considered fall in the broad categories of broadcasting, clustering, monitoring, and topology control of WSNs.

We mainly focus on devising *distributed* (and *local*) algorithms for these problems, where individual nodes execute algorithms on their own to compute solutions to a problem of a global nature. A distributed algorithm is one where nodes individually execute the same algorithm and make decisions accordingly without knowing the global topology of the network. However, in some distributed algorithms, it is allowed that nodes may know a little global information (e.g., the number of sensors in the network and/or the maximum degree of the underlying graph). A more strict version of the distributed algorithm is known as the local algorithm. Informally, in a local

algorithm, a node is allowed to communicate only with its neighbors, which are at most a constant *hop* away from it, to make decisions during the execution of the algorithm (more about local algorithms is described later in this chapter).

Unlike centralized systems, the distributed and local approaches relieve sensors from sending their information (e.g., ids, geographic positions, neighborhood information, covered areas, and so on) to a central base station which runs the algorithm and returns the results to the sensors. Sending information to a base station is energy-consuming and generally not scalable. However, we consider the following problems in the thesis:

1.2.1 Broadcasting Issues

Broadcasting is an operation by which a message, generated by a node in the network, is forwarded to all other nodes in the network. Following a simple flooding approach to do this is inefficient. This is because a lot of unnecessary message transmissions (or just *transmissions*) are generated and relayed in the network, which in turn, causes nodes to dissipate their valuable energy quickly. Thus we need to devise energy-efficient algorithms that can avoid or at least reduce the amount of redundant transmissions.

Virtual backbone

Establishing a *virtual backbone* in WSNs, which can be seen as an analogue of the fixed communication infrastructure in wired networks, helps reduce the number of retransmissions of the energy-constrained sensors. In a virtual backbone, only a "small" number of nodes in the network are engaged in transmissions. Basically, a

virtual backbone consists of a subset of nodes in the network which are responsible for relaying messages throughout the network. The *connected dominating set* (CDS) is one of the earliest and most popular structures proposed as a candidate for a virtual backbone in wireless networks [18]. Since the work of Das et. al. [18], an extensive research effort has been put into finding small-sized CDSs in WSNs. The smaller the size of a CDS, the fewer the number of transmissions in the network. Chapter 3 studies the problem of computing CDSs in a distributed way and discusses other relevant issues.

Family of Connected Dominating Sets

We make an important observation regarding finding small-sized or even optimal CDSs in WSNs. Although the general focus of research for broadcasting algorithms is to obtain small cardinality CDSs, generating a single CDS (even if it is optimal) and using it all the time causes the nodes in the CDS to run out their energy faster than other nodes (which are not in the CDS). This is because the nodes in the CDS are responsible for forwarding messsages in the network on behalf of other nodes, whenever any node in the network has some messages to broadcast. Considering this important aspect, we study the problem of computing a family of CDSs in a distributed manner such that only one CDS is *active* for a certain period of time and other nodes can be put in the energy-efficient *sleep* mode.

Thus, using the CDSs iteratively, we can expect to substantially reduce the energy consumption of the individual nodes by repeatedly switching them back and forth from the active to the sleep mode and make the *lifetime* of the network longer. By the lifetime of a WSN, we mean the elapsed time between the deployment of the

sensors and the time when the first sensor runs out of energy. Keeping this in mind, we aim at studying the problem of locally generating a family of non-trivial CDSs in WSNs with the goal to minimize the number of participations of individual nodes in these sets. Chapter 4 deals with this problem and investigates special cases where the algorithm achieves 'good' results.

1.2.2 Clustering

Clustering is a heavily studied topic [26, 21, 22, 27, 53, 4, 74] in the WSNs community, where the goal is to divide the whole network in a number of clusters (not necessarily disjoint) and select a node as a cluster head (CH) from each of the clusters. Each such CH is supposed to be active and do all the coordination works, e.g., sensing, data gathering, and transmitting data on behalf of the cluster to the base station, while the remaining nodes in the cluster can go into the sleep mode. One of the basic problems in clustering is to minimize the number of CHs under the condition that for any node in the network either it is a CH or directly connected to at least one CH. This would leave most of sensors in the energy-efficient sleep mode. This problem is also known as the *minimum dominating set* problem.

However, as the set of CHs (even the minimum set of CHs) is busy all the time for sensing, processsing, and transmitting data, they quickly run out of energy, while other nodes (which are not CHs) are left with considerable amount of energy. This causes a significant imbalance in the energy reserve in the nodes and reduces the network lifetime. One possible way to overcome this situation is to find a family of disjoint sets of CHs and make them active iteratively such that energy consumption is balanced among the nodes in the network.

Disjoint Dominating Sets

In the context of WSNs (considering the above circumstance), we address the following problem: "Given a WSN, find the maximum number of disjoint dominating sets". A dominating set of a graph is a subset of nodes such any node in the graph either is in the subset or a neighbor of some node in the subset. Finding the maximum number of disjoint dominating sets in a graph is also known as the *Domatic Partition* problem. Although the domatic partition problem is NP-hard in general graphs, it is unknown whether the same is true for *unit disk graph*s. Chapter 5 discusses the problem of finding the maximum number of disjoint dominating sets in WSNs (a WSN is modeled as a unit disk graph) and presents a centralized algorithm to solve the problem.

1.2.3 Target Monitoring

Monitoring (also called *coverage*) is one of the important and extensively studied topics in sensor networks [8, 10, 11, 47, 31, 71, 78]. In general, the main goal of research in this line is to devise scheduling algorithms such that individual sensors in the network are assigned time-slots which indicate to them during which time-slots they will be active and during which time-slots they will sleep. Given a WSN which monitors certain targets, it is sometimes possible to find only a subset of the sensors and engage them to do the same monitoring activity. Thus, instead of making all nodes active for this purpose (which is obviously redundant), we can choose possibly a small subset which can guarantee the same monitoring. This observation leads researchers to devise efficient algorithms such that at any time only a small number of nodes are made active to monitor all the targets in question.

Chapter 6 investigates the following target monitoring problem. Given a set of stationary targets T and a set V of sensors, the problem asks for generating a family of subsets of sensors called the *monitoring sets*, such that each monitoring set monitors all targets. The monitoring sets are iteratively made active in order to provide continuous monitoring to the targets. The objective of this problem is to maximize the number of monitoring sets and minimize the maximum number of participations of a sensor in these monitoring sets. This chapter provides a local technique to solve the problem approximately, discusses theoretical results, and compares experimental results with the theoretical bounds.

1.2.4 Self Protecting WSNs

We study an interesting problem which deals with providing sensors with a level of protection by other sensors. Since sensors provide monitoring to targets, it is often necessary to give them a level of protection (by other sensors) such that sensors can take certain actions when attacks are targeted on them. One natural idea is to monitor sensors by their neighbors such that the neighbors can inform the base station when other sensors are in danger (or not working due to malfunction). We elaborate on this in the following.

Sometimes it may be required to know whether every sensor in the network is in good health to render its tasks. Since a *bad* sensor, i.e., a malfunctioning or dying out sensor cannot inform the base station of its condition, the targets monitored by the bad sensor become unprotected and the system has no way to know about this vulnerability. In that case, we need to find a subset of sensors whose function is to monitor other sensors, such that when any sensor (including themselves) fails or

cannot function properly these sensors will notify the base station of the situation. The base station then takes appropriate actions, such as, deploying additional nodes in replacement of the non-functioning ones to provide continuous protection to the targets.

Keeping this in mind, the *p-self-protection subset* problem has been formally introduced in [79]. The p-self-protection subset problem is defined to be a subset of nodes such that for any node in the network there are at least p nodes from the subset that monitor it. Chapter 7 studies the problem of p-self-protection ($p \geq 1$). That chapter begins by giving a counterexample to the algorithm [77], and then provides a fast and distributed algorithm to approximately solve the p-self-protection problem.

1.2.5 Topology Control Problems

Topology control is one of the pivotal problems which has received a lot of attention due to its numerous applications in WSNs such as coverage, connectivity, and routing [44, 45, 48, 75, 84, 50, 49, 63, 65, 64, 80, 81, 19, 32, 6]. The main idea is to obtain a subgraph from the underlying network by removing some (possibly redundant) links in order to enhance the overall performance of the network in some ways.

Topology control algorithms, in general, deal with finding a suitable structure (subgraph) of the given graph. This resulting subgraph is expected to have certain features (e.g., *connectedness, planarity, sparseness, bounded-degree, and spanner property*, etc.) which mainly facilitate routing and other decisions in the network.

Finding a topology with the above features has certain advantages. First, a number of routing algorithms including the well-known GPSR routing algorithm [12] can only be applied if and only if the network topology is planar. Second, by eliminating a

number of links, we can substantially reduce the burden of the nodes by letting them keep only a subset (possibly small) of their neighbors. The smaller the neighborhood, the faster the information processing of a node. Having a few neighbors also causes a small number of transmissions and helps prolong the lifetime of the network by limiting the number of retransmissions. Topology algorithms tend to remove longer links in the network and keep smaller ones since long links consume more power than shorter links for transmission (recall that the power spent on a link for a transmission is directly proportional to at least the square of the length of that link). This saves energy in the long run for routing and thus helps extend the network's lifetime.

Chapter 8 investigates the topology control problem. It begins by showing cases where a slight error in the exact geographic position, and/or the distances between neighboring sensors, can make algorithms yield unwanted results (e.g, a disconnected subgraph). A robust and resilient local algorithm with realistic assumptions is presented in the chapter, which is supposed to provide an 'acceptable' performance in average cases. This chapter provides theoretical and experimental results.

1.2.6 Contribution of the Thesis

As centralized algorithms may not be suitable in large WSNs due to scalability and energy issues, our aim in this thesis is to design energy-efficient distributed and local algorithms to address the aforementioned problems. We devise distributed (or local) solutions which are fast, scalable, and can be easily implemented in WSNs for the problems except for the domatic partition problem, where we provide a centralized algorithm.

Thus the contributions of the thesis are:

1. Designing and analyzing a fast, distributed and constant-approximation algorithm for the connected dominating set problem.

2. Solving the energy consumption issue of the sensors in a balanced way by generating a set of connected dominating sets in a distributed fashion.

3. Developing a technique (centralized) for the problem of approximately finding the maximum number of dominating sets in unit disk graphs.

4. Presenting a local solution to the target monitoring problem.

5. Providing a distributed constant-approximation algorithm for the self-protection problem.

6. Introducing a fast, easily implementable local algorithm for the topology control problem in WSNs.

1.3 Outline of the Thesis

The rest of the thesis is organized as follows. In Chapter 2 we provide definitions and assumptions that are used throughout the thesis. Chapter 3 discusses the CDS problem and presents an algorithm to solve it. As a continuation of this problem, Chapter 4 begins with an example that sheds light on the issue of having multiple CDSs instead of just a single CDS and shows a way of generating a family of different CDSs. It also explores some interesting relationships between *vertex-cuts* and the number of CDSs of a graph. The clustering problem, specifically the domatic partition problem is studied in Chapter 5 and a centralized method is presented to solve the problem. In Chapter 6, we define the target monitoring problem and give

a local algorithm towards solving it. The self-protection in WSNs is described and a distributed technique to handle the problem is provided in Chapter 7. Chapter 8 is devoted to discussing the topology control problem which begins by showing a counterexample to some previous work and presents a fast local algorithm for it. Finally, Chapter 9 provides a summary of the results and concludes with a number of open problems.

In this thesis, I do not include a separate "Previous Work" chapter since each chapter will include a section concerning previous work on the problem addressed in that chapter.

Chapter 2

Definitions and Model

In this chapter, we define some terms which will be used thoroughout the thesis. However, there are certain terms and notations which are defined in the corresponding chapters according to their relevance.

2.0.1 Graph Terminology and other

We begin with some terminology regarding graphs. Assume a set of sensors are deployed randomly and uniformly in the plane. We model the wireless network formed by the sensors as a geometric undirected graph $G = (V, E)$, where $V = \{v_1, v_2, \cdots, v_n\}$ denotes the set nodes (sensors) and there is a link between two nodes v_i and v_j, $v_iv_j \in E$, if the Euclidean distance between them is at most 1. In other words, G is known as a *unit disk graph (UDG)*. Unless otherwise mentioned, henceforth we assume G is a UDG and connected and the number of nodes in G is denoted by $n = |V|$.

The *hop-distance* between two nodes v_i and v_j in G is the minimum number of

edges between them and denoted by $d(v_i, v_j)$. Also $d(v_i, v_j)$ refers to the length of the shortest path between v_i and v_j. The *diameter* $Diam(G)$ is defined as the maximal hop-distance between any two nodes in G, i.e., $Diam(G)$=$\max_{u,v \in V} d(u, v)$. If it is clear from the context then we use distance instead of hop-distance.

The *neighborhood* of a node $u \in V$, denoted by $N(u)$, is the set of nodes that are u's direct neighbors (not including u) in G. We use $N[u]$ to include u in the neighbor set. Formally, the neighborhoods of u are defined as

$$\begin{aligned} N(u) &= \{v | uv \in E\} \\ N[u] &= N(u) \cup \{u\}. \end{aligned}$$

In the distributed setting, we extend the definition of neighborhood in some depth. For some constant integer $k \geq 1$, the k-neighborhood $N_k(u)$ of a node u is defined as the set of all nodes which are at most k-hops away from u. That is,

$$N_k(u) = \{v | d(u, v) \leq k\}.$$

Similarly $N_k[u]$ is defined. For simplicity, we use $N(u) = N_1(u)$. The degree $deg(u)$ of node u is the number of direct neighbors it has, i.e., $deg(u) = |N(u)|$. The maximal and minimal degrees in G are defined as Δ =$\max_{u \in V} deg(u)$ and δ =$\min_{u \in V} deg(u)$, respectively. Each node u has a unique integer id, $id(u)$ of size $O(\log n)$.

We call two nodes $u, v \in V$ independent if $uv \notin E$. A subset $I \subseteq V$ is called an independent set if all pairs of nodes in I are independent. Set I is called a *maximal independent set* (MIS) if no independent set $T \supset I$ exists. A subset $D \subseteq V$ is a

dominating set if each node $u \in V$ is in D or adjacent to some node $v \in D$. If D induces a connected subgraph, then D is called a *connected dominating set* (CDS). A CDS with minimum cardinality is called a *minimum connected dominating set* (MCDS).

In this thesis, we study some combinatorial optimization problems for which we provide distributed algorithms. Since most combinatorial optimization problems are NP-hard, polynomial-time algorithms designed to solve them provide only approximate solutions. The quality of an approximate solution can be measured by comparing it with the optimal solution. Consider a minimization problem. Let ALG_I denote the value of the solution generated by algorithm A for all inputs I. If OPT_I is the value of the optimal solution for I, then the *approximation factor (or ratio)* α of algorithm A is given by,

$$\alpha = \max_I \frac{ALG_I}{OPT_I}$$

for all possible input I. For a maximization problem, the approximation ratio is similarly defined as α=max$_I$ $\frac{OPT_I}{ALG_I}$.

Finally, we use the notation $\log^* n$, which is the number of times the logarithm function must be iteratively applied before the result is less than or equal to 1.

2.0.2 Computational Model

Most distributed systems consist of a set of nodes which are interconnected in some way that permits them to cooperatively exchange information among nodes in order to solve certain problems. Note that distributed systems are the opposite of the centralized system where a single entity, knowing the whole topology of the system,

is responsible for solving the problem. However, in a distributed system each of the nodes has a certain degree of autonomy allowing them to execute their own protocol, share the results with neighbors to produce solutions for some applications. There are two basic models that form the basis of most distributed systems, namely the *message passing* and the *shared memory* models [61]. In the shared memory system, nodes do not communicate with each other directly. Instead, the model assumes the existence of a common memory storing variables that are shared (read and write) by all nodes in the system. In contrast, the message passing model treats communications explicitly. Whenever a node wants to speak with another, it must send a message to its neighbors which may relay the message to their neighbors and so on to eventually reach the intended receiver. The information is sent and received using the underlying communication media (wired or wireless).

In the thesis, we follow the message passing model. We assume communication to be synchronous for all the distributed and local algorithms. In synchronous systems, time is divided into rounds and in each round every node can do the following three things: it can send a unique message to each of its neighbors, receive messages from its neighbors, and perform some local computation based on the information received in previous rounds. Local computation is assumed to take negligible time as compared to the message transmission time, so in synchronous systems, local computation time is reasonably small [61]. Thus the *time complexity* of a synchronous distributed algorithm is defined as the number of rounds until all nodes terminate. In general, all proposed algorithms in synchronous systems neglect local computation time (assuming this model allows nodes to compute a 'computable' function) and derive a time complexity bound by the number of communication rounds it takes in the

worst-case execution. Similarly the *message complexity* of a synchronous distributed algorithm is defined as the total number of messages generated by the algorithm in the worst-case scenario.

The size of a message (that is, the number of bits contained in each of them) in distributed synchronous systems plays an important role in message passing models. The question of message sizes gives rise to two fundamental limitations in message passing networks, namely, $\mathcal{LOCAL}$ and $\mathcal{CONGEST}$ [54, 59, 61] which differ in the way of how much information (bits) should be included in a single message. We elaborate on them in the following.

2.0.3 Local Algorithms

The $\mathcal{LOCAL}$ model, also known as Linial's free model [54], places no restriction on the size of a single message, that is, a node can send an arbitrarily large message to its neighbors in any communication round. This model also assumes that communication is synchronous and that all the nodes wake up simultaneously and start the execution of the protocol at the same round. Since the message size is unlimited, congestion or bottlenecks have no influence on an algorithm's time or message complexity. However, one powerful aspect of the $\mathcal{LOCAL}$ model is that a node in some distributed algorithm is able to know its k-neighborhood exactly in k communication rounds. Since the message size is unbounded, a node can have all information (e.g., ids and interconnections) of all nodes which are at most k-hops away in k rounds. Collecting the complete k-hop neighborhood is possible when each node sends its complete information (in its arbitrary long single message) to all neighbors at every round. This model gives rise to the definition of *local* algorithms. A distributed

algorithm is called a $k - local$ algorithm if

1. each node collects its k-neighborhood information (ids, and neighborhood information) in k communication rounds and

2. it produces the output by performing only local computation with the information collected (no communication required). (Recall that local computation time is negligible) [57].

In the $\mathcal{LOCAL}$ model, every computable problem can be solved in $Diam(G)$ time since this is the time required for any node to learn the entire topology of the underlying graph.

On the other hand, the goal of the $\mathcal{CONGEST}$ model is to limit the message size and consider the aspects of congestion and *volume of communications*. Since the $\mathcal{LOCAL}$ model abstracts away some difficulties and challenging issues arising in the networks by making the message size long, the $\mathcal{CONGEST}$ model tries to capture the issues of congestion and bottlenecks by limiting the message size to $\mathrm{O}(\log n)$ bits. That is, every node $u \in V$ can send a message of size $\mathrm{O}(\log n)$ bits to each of its neighbors $v \in N(u)$ at every round. Because of this restriction, each node is now able to send at most a constant number of node identifiers in each message to its neighbors. Given this limited message size, many problems on the complete network graph (e.g., the Minimum Spanning Tree) which can be solved optimally relatively easily in the $\mathcal{LOCAL}$ model becomes non-trivial in this model [57].

In the thesis, we will consider the $\mathcal{LOCAL}$ model (synchronous model with unbounded messages and local computations). As described before, in the $\mathcal{LOCAL}$

model, at each round a node can send a unique message to each of its neighbors. However, in the context of WSNs, when a node sends a message, it is broadcast to all its immediate neighbors. That is, in WSNs, instead of sending a separate message to each of its neighbors, a node sends a single message to all its neighbors. Thus our model (we name it, $\mathcal{LOCAL}_S$) does not deviate from the $\mathcal{LOCAL}$ model and the computational powers of $\mathcal{LOCAL}$ and $\mathcal{LOCAL}_S$ are the same.

Chapter 3

Connected Dominating Sets

Connected dominating sets (CDSs) [13, 28, 16, 15, 56, 5, 68, 82, 70, 47, 83, 37] are probably the most common way of constructing virtual backbones for broadcasting operation in WSNs. This is because such backbones are guaranteed to reduce unnecessary message transmissions or flooding in the network. In this chapter we propose a simple distributed algorithm to construct a *small*-sized CDS. Considering the nodes deployed in the plane, our main idea is based on the computation of *convex hull*s of nodes (nodes are considered points in the plane) in a distributed manner and a simple coloring scheme, which produces a CDS in UDGs whose size is at most $50.54*|\text{MCDS}|$, where $|\text{MCDS}|$ is the size of a minimum CDS. To the best of our knowledge, this is a significant improvement over the best published results in reducing the approximation factor [13]. We also analyze geometric grids and trees to compute the exact approximation ratios for the problem and show that the algorithm produces an MCDS if the graph is a tree and in the case of grids the approximation factor is 2.

The rest of the chapter is organized as follows. Section 3.1 presents related work. In Section 3.2 we provide definitions and assumptions that are used throughout the

chapter. The main idea of the distributed algorithm is described in Section 3.3. We analyze the algorithm and give some theoretical results including time and message complexities of the algorithm in Section 3.4. We conclude in Section 3.5.

3.1 Related Work

Guha and Khuller [28] studied the MCDS problem and showed that this problem is NP-hard in arbitrary undirected graphs. They present a centralized approximation algorithm which guarantees a $O(\log n)$ approximation factor, where n is the number of nodes in the graph. Later it was shown in [16] that computing an MCDS is NP-hard even for UDGs. The first centralized method towards solving this problem achieves a 10-approximation factor and was given by Marathe [56]. Cheng *et. al* [15] presented a polynomial-time approximation scheme that guarantees an approximation factor of $(1 + 1/s)$ with running time of $n^{O((s \log s)^2)}$, $s > 0$. However, in reality, centralized algorithms may not be preferred for large networks because of their power consumption. Therefore, distributed algorithms are sought to find an approximate MCDS in such networks. Several distributed approximation algorithms exist in the literature, some of the earlier attempts include the works of Stojmenovic and Xiang-Li [70, 47, 83] which distributedly construct a CDS of size $n/2$ times that of an MCDS.

Alzoubi *et al.* [2] proposed the first distributed algorithm guaranteeing a constant approximation factor for the CDS based on MIS in UDGs. The main features of their distributed algorithm include an approximation factor of 8, $\mathrm{O}(n)$ time complexity, and $\mathrm{O}(n \log n)$ message complexity. One of the important results established in that paper, is an $\Omega(n \log n)$ a lower bound on the message complexity of any distributed algorithm for nontrivial CDSs.

However, the algorithm in [2] suffers from the construction of a spanning tree in wireless networks, which is expensive since the time complexity is $Diam(G)$ in which case a node "needs to" know the whole topology of the network. Building a spanning tree and maintaining a CDS in large and volatile WSNs with common node and link failures incur a significant number of retransmissions and communication overhead. Later, Alzoubi *et al.* came up with a constant factor distributed algorithm [1] that does not rely on the construction of spanning trees. However, the algorithm produces a CDS with size at most $192*|\text{MCDS}|+48$. Lately, Funke et al. [23] gave a distributed algorithm which provides an approximation factor of 6.91 and outperforms all other approximation factors of previous distributed algorithms. Although the factor is small, the time complexity of their algorithm is still $Diam(G)$, which is $O(n)$ in the worst-case.

In an attempt to obtain a small constant factor, a distributed algorithm is proposed in [51] which produces a CDS with size at most $172 * |\text{MCDS}| + 33$ in UDGs. Subsequently, the next level of reduction in the constant factor was achieved by the distributed algorithm in [14], where the factor is 147 (specifically, $147*|\text{MCDS}|+43$). Recently D. Chen *et al.* managed to achieve even a smaller constant factor by a simple distributed algorithm. The cardinality of the CDS produced by their algorithm is at most $76 * |\text{MCDS}| + 19$. Here, we present a technique that reduces this constant factor to 50.54 that is, it produces a CDS with size at most $50.54 * |\text{MCDS}|$. We analyze special classes of graphs, for example, trees and grids, where the approximation ratios obtained by our algorithm are 1 (optimal) and 2, respectively.

3.2 Some Definitions

Recall that nodes are deployed in the plane and the network is modeled by an undirected graph $G = (V, E)$, where a link $uv \in E$ between two nodes $u, v \in V$ exists if they are within their transmission range, which is normalized to 1. We use a computational geometry idea called the *convex hull* in our algorithm. The convex hull of a set of points in the plane is defined as the smallest convex polygon that includes all the points in the set such that each point is either a corner of the convex hull or lies inside it. The convex hull of $N(u)$ is denoted by $CH(N(u))$. We say $CH(P)$ of some point set $P \subseteq V$ is *completely included* (or just *included*) in $CH(Q)$ of point set $Q \subseteq V$ if $CH(P) \subseteq CH(Q)$.

3.3 A Distributed Algorithm for CDS

In this section we present a simple algorithm for computing a CDS in G based on the computation of convex-hulls of the 1-hop neighborhood of individual nodes and a coloring scheme of the nodes of G. There are three colors, namely 'black', 'grey' and 'white'. Initially all nodes are white.

Here we outline the algorithm (call our algorithm A):
The algorithm consists of two phases (Phase I and Phase II). In Phase I, we distributedly form a primary CDS denoted by B, which is pruned in the next phase (II) to generate the final CDS denoted by CDS* (CDS* is the solution we are looking for). After the execution of Phase I, all nodes change their respective colors and become either black (if the node belongs to B) or grey (otherwise).

In Phase II, first we prune the set B of black nodes. In order to do so, we apply a

distributed MIS algorithm in B. Let the set of MIS nodes returned by the algorithm be $S \subseteq B$. Although an MIS is a dominating set in a graph, the subset S is not a dominating set in G since S is obtained from the nodes in B (not from the nodes V). Then we form a dominating set in G consisting of nodes in S and a set B' of *additional* nodes ($B' \subseteq (V \setminus S)$). Thereafter, a set C of connectors is selected from $V \setminus S$ to connect the nodes in S such that $S \cup B' \cup C$ forms the desired CDS*. Following is a detailed description of the two phases of algorithm A:

Phase I (Coloring nodes):

Initially all nodes are white. Each node u locally computes the convex hull $CH(N[u])$ following an optimal convex hull algorithm [17]. Then u compares $CH(N[u])$ with $CH(N[v])$ for all $v \in N(u)$ and colors itself in the following way:
(1) **If** there is no v such that $CH(N[u]) \subset CH(N[v])$ **Then** u colors itself black.
(2) **If** there is some v such that $CH(N[u]) \subset CH(N[v])$ **Then** u colors itself grey.
(3) **If** there are some v's such that $CH(N[u]) = CH(N[v])$ **Then** node u colors itself black **If** it has the smallest id among all such v's, otherwise it colors itself grey.

This phase colors every white node either black or grey. We show that that set of black nodes B forms a CDS in G (Lemma 3.4.1).

In Figure 3.1, we give a simple example where the black nodes by Phase I of the algorithm A form a CDS. Node 4 colors itself black since its convex hull $CH(N[4]) = \{1, 2, 3, 5\}$ (shown in dashed line segments) is not included in any of the convex hulls

$CH(N[1])$, $CH(N[2])$, $CH(N[3])$ or $CH(N[5])$. For the same reason, nodes 5 and 6 color themselves black. However, nodes 1, 2, 3, 7, 8, and 9 are colored grey because each of their computed convex hulls is included in its corresponding neighbor's convex hull.

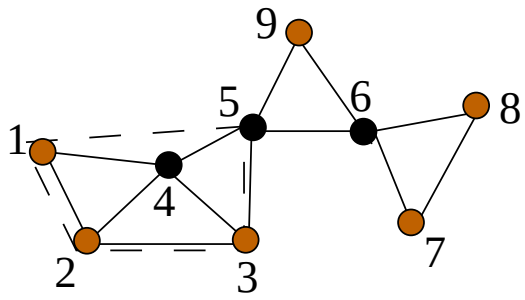


Figure 3.1: Node 4 is black since $CH(N[4]) = \{1, 2, 3, 5\}$ (shown in dashed line segments) is not included in any of $CH(N[1])$, $CH(N[2])$, $CH(N[3])$ or $CH(N[5])$. The same applies to nodes 5 and 6.

Phase II (Finding an MIS S and a set C of connectors):

This phase begins when Phase I is finished, that is, when there are no white nodes in G. In this phase, first, as mentioned before an MIS S is constructed from the set of black nodes B by following the distributed algorithm [66]. The authors in [66] present a very fast $O(\log^* n)$ distributed algorithm for computing such an MIS for any *growth bounded* graph, where the growth boundedness of a graph means that in a constant neighborhood of any node there is a constant number of independent nodes. (It can be mentioned that a UDG is a growth bounded graph since for any node u the number of independent nodes in its r-neighborhood, $N_r(u)$, is $O(r^2)$.)

As S is not a dominating set in G, some grey nodes in G may not be dominated (covered) by S. Note that according to the algorithm, for each grey node u there must

be at least a black neighbor $v \in N(u) \cap B$ such that $CH(u) \subseteq CH(v)$, otherwise u would not be grey. Let $B(u)$ denote the set of such black neighbors of u.

Now we find an additional set B' of nodes from $V \setminus S$ in the following way to make $S \cup B'$ a dominating set in G. Call the grey nodes not dominated by S *orphan* nodes. An orphan node u selects a black node from $B(u)$ to dominate itself. If $|B(u)| > 1$, then u selects the one with the minimum id. The set of black nodes selected by all such grey nodes constitutes the set B'. Observe that $S \cup B'$ is now a dominating set in G because all the orphan nodes have at least one neighbor in B'. Change the color of the nodes in $B \setminus (S \cup B')$ into grey.

Although $S \cup B'$ dominates the nodes in G, it is not necessarily connected. Therefore, a set of connectors C from $V \setminus S$ is sought to connect the MIS nodes S in order to find a CDS in G. A node $u \in S$ connects to a node $v \in S$, $u \neq v$ with the minimum id, which is at most 3 hops away. Therefore, at most two intermediate nodes between u and v are selected to be in C. All such selected intermediate nodes form the connector set C. The nodes in C color themselves black. Finally, the union $S \cup B' \cup C$ forms the CDS* in G.

For an illustration, we provide a complete example of Phase I and Phase II of algorithm A is given in Figure 3.2:

3.4 Algorithm and Analysis

The proposed distributed algorithm is given in Figure 3.3 which is executed at each node u.

As mentioned before, the first phase of the algorithm is finished when each node

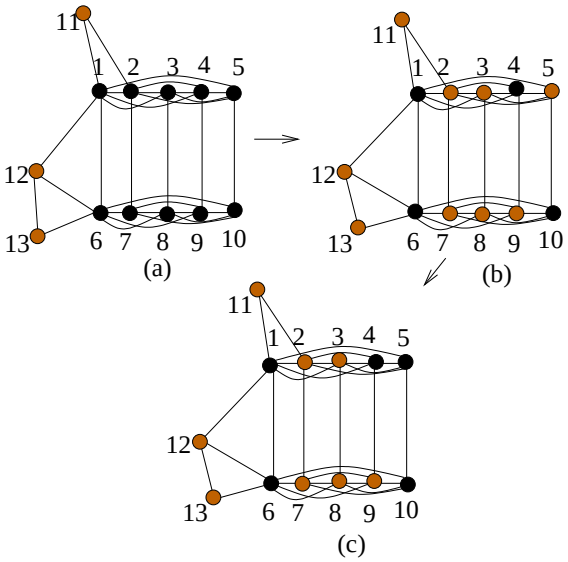


Figure 3.2: (a) After Phase I, $B = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$. (b) S consists of nodes $\{4, 10\}$. Additional node set is $B' = \{1, 6\}$. (c) After Phase II the solution is $S \cup C \cup B' = \{1, 4, 5, 6, 10\}$, where $C = \{5\}$. The MCDS consists of $\{1, 6\}$.

is either black or grey. Here we provide a lemma which is essential for the proofs of other results. This lemma guarantees that the set of black nodes B found through Phase I is a CDS.

Lemma 3.4.1 *The set B forms a CDS in G.*

Proof We prove the lemma by contradiction. Suppose u is grey. Then from the algorithm (consider Phase I only) we derive either **Case 1**: $CH(N[u]) \subset CH(N[v])$ for some $v \in N[u]$) or **Case 2**: if such v does not exist, there are some nodes v with $CH(N[u]) = CH(N[v])$ and there is a v with $id(v) < id(u)$. In Case 1, if there are several such nodes with this property, take a node v with the maximal size (in terms of the number of vertices) $CH(N[v])$ (using the partial order "subset of or equal"). If there is still a tie, take the one with the minimum id. Node v cannot be colored grey

Distributed Algorithm for CDS
PHASE I:
1: **If** $\exists v \in N(u)$ such that $CH(N[u]) \subset CH(N[v])$ **Then**
2: Color u grey **Endif**
3: **If** $\nexists v \in N(u)$ such that $CH(N[u]) \subset CH(N[v])$ **Then**
4: Color u black **Endif**
5: **If** $\exists v \in N(u)$ such that $CH(N[u]) = CH(N[v])$ **Then**
6: **If** $id(u) < id(v), \forall v \in N(u)$ **Then**
7: Color u black
8: **Else**
9: Color u grey
10: **Endif Endif**
PHASE II:
11: **If** u is black **Then** Run MIS [66] **Endif**
12: **If** u is grey **Then** Select a node in $B(u)$ **Endif**
13: **If** u is in S **Then** Select connectors between u and v **Endif**
// $v \in S$ is at most 3 hops away and $id(u) > id(v)$

Figure 3.3: A Distributed algorithm for the CDS problem.

at the beginning of the algorithm since this would mean the presence of a node w, where $w \in N(u)$ and $CH(N[u]) \subset CH(N[v]) \subset CH(N[w])$ implying that $CH(N[v])$ is not maximal. Also it can not be colored grey in the next step since that implies that there is a node w with $CH(N[u]) \subset CH(N[v]) = CH(N[w])$ and $id(w) < id(v)$, so v does not have the minimum id. Therefore, v is colored black.

Of all such nodes (v) with the property of Case 2, take the one with the minimum id. Node v is not colored grey at the beginning of the algorithm, since this would imply the presence of a node w with $CH(N[u]) = CH(N[v]) \subset CH(N[w])$ and $w \in N(v)$. However, it will be colored black in the next step but this is a contradiction. So B is a dominating set.

In order to prove that the node set B is connected, we show that there is a path between any two nodes $u, v \in B$ containing only black nodes. For the sake

of contradiction assume such a path, $\pi = (u, \cdots, v_{i-1}, v_i, v_{i+1}, \cdots, v)$ contains a grey node $v_i \notin B$. Here the node that precedes v_i is labeled as v_{i-1} and the one that follows v_i is labeled as v_{i+1} on the path. Node v_i is colored grey since it has some neighbor $v_t \in N(v_i)$ (v_i is not an isolated node and has at least one neighbor) such that $CH(N[v_i]) \subseteq CH(N[v_t])$, where either $id(v_t) < id(v_i)$ and $CH(N[v_i]) = CH(N[v_t])$ or $CH(N[v_i]) \subset CH(N[v_t])$. If that is the case then v_t's neighborhood must include all of v_i's neighbors, that is, $v_{i-1}, v_{i+1} \in N(v_t)$. Thus instead of taking v_i in π, we can include black node v_t in π. If there are other grey nodes in the path π, then we can apply the same argument for all such grey nodes in the path. So the path $\pi = (u, \cdots, v_{i-1}, v_t, v_{i+1}, \cdots, v)$ contains only black nodes which is a contradiction to the original assumption that π contains some grey node. ∎

Since B is connected, we can apply the distributed MIS algorithm [66] to B which takes $O(\log^* |B|)$ time to compute an MIS S in B. We will prove that the cardinality of the union of S and a set of additional nodes, B' (i.e., $|S \cup B'|$) is at most 2.5 times the size of an MCDS of G.

Before that we need the following lemma:

Lemma 3.4.2 $|S| \leq 1.66 * |MCDS| + 2$

Proof Consider a node u in the MCDS and its 2-hop neighborhood $N_2[u]$ such that for any node $v \in N_2[u]$ we have $N[v] \subseteq N_2[u]$ (see Figure 3.4). In other words, v's 1-hop neighborhood is contained in $N_2[u]$.

Node u can have at most 5 MIS nodes, labeled u_i, $1 \leq i \leq 5$ as shown, in its 1-hop neighborhood. A node $v \in N_2[u]$ in the 2-hop neighborhood can be colored black if v is adjacent to both u_i and u_{i+1} (assume modulo arithmetic). This is because

v's convex hull $CH(N[v])$ will not be included in any of its neighbors' convex hulls. Similarly, any node v^* which is 2-hop away from u and not adjacent to both u_i and u_{i+1} will not be colored black since $CH(N[v^*])$ must be included in the convex hull $CH(N[u^*])$ of some node $u^* \in N[u]$.

According to the above arrangement of the nodes we can have at most 11 black nodes in $N_2[u]$. As we apply an MIS algorithm, we can have at most six MIS nodes in $N_2[u]$ (the MIS nodes are labeled '1' in the figure).

Now we determine the minimum number of MCDS nodes that u must have in $N[u]$ in order to cover all the MIS nodes in $N_2[u]$. In order to do that we need to investigate the following four cases:

Case 1: At least four MCDS nodes, namely, u, u_i, u_{i+2} and u_{i+3} are required to cover (dominate) at most six MIS nodes (Figure 3.4(a)).

Case 2: At least three MCDS nodes (u, u_i, u_{i+2}) are required to cover at most five MIS nodes (Figure 3.4(b)).

Case 3: At least two MCDS nodes (u, u_i) are required to cover at most three MIS nodes (Figure 3.4(c)).

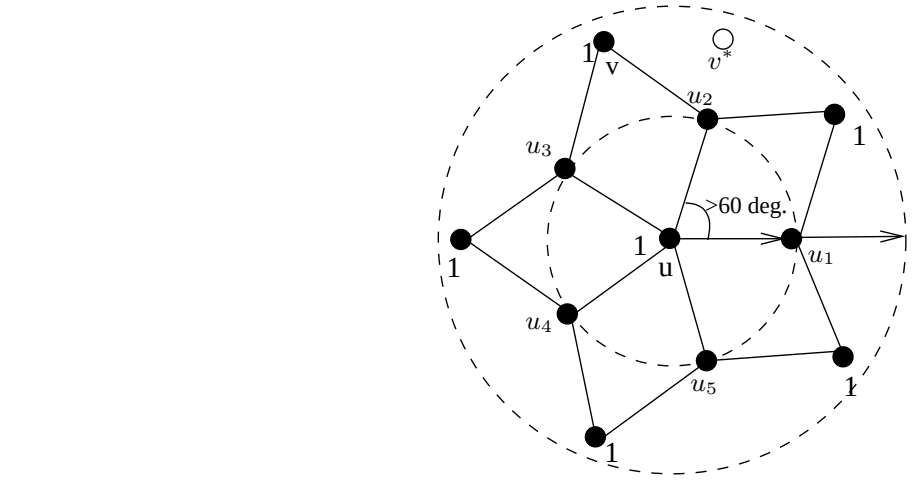
Case 4: At least one MCDS node (u) is required to cover one MIS node (Figure 3.4(d)).

Analyzing the above relations we have the following:

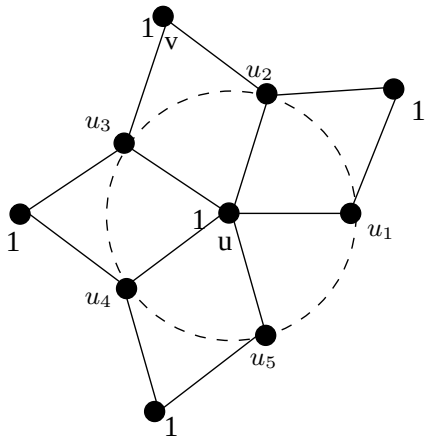
Thus $|S| \leq 5 * \lceil \frac{|MCDS|}{3} \rceil \leq \lceil 1.66 * |\text{MCDS}| \rceil + 1 \leq 1.66 * |\text{MCDS}| + 2$. This completes our proof. ∎

Theorem 3.4.3 *Let B be a CDS in G and S be any MIS in B. If B' denotes the set of additional nodes, then $|S \cup B'| \leq 2.66 * |MCDS| + 2$.*

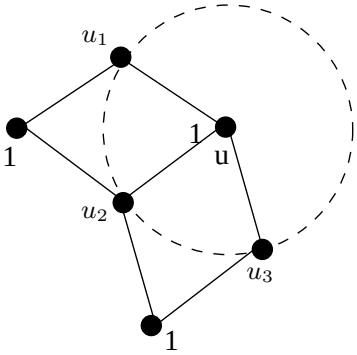
Proof Recall that for any orphan node u (u is grey) there must be at least a black



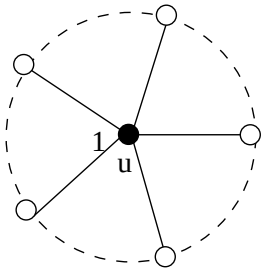
(a) Case 1



(b) Case 2



(c) Case 3



(d) Case 4

Figure 3.4: Proof of Lemma 3.4.2.

node $v \in N(u)$ such that $CH(N(u)) \subseteq CH(N(v))$, otherwise u would not be grey. In order for the MCDS to cover u, either u must belong to an MCDS or there will be at least a node $v' \in \text{MCDS} \cap N(u)$. In any case an MCDS must have at least one node from $N[u]$. Since u selects the node with the minimum id from $N(u)$, the number of nodes in MCDS will not be greater than the number of nodes selected by all orphan nodes in the graph. Therefore, $|B'| \leq |\text{MCDS}|$, where B' is the set of nodes selected by all orphan nodes in the graph.

Therefore, combining the results from Lemma 3.4.2 we obtain the following, $|S \cup B'| \leq 2.66 * |\text{MCDS}| + 2$. ∎

Now we have $S \cup B'$ black nodes in G and all other nodes are dominated by them. Recall that $B \setminus (S \cup B')$ are colored grey. As $S \cup B'$ may not be a CDS, we find a set C of connectors to connect the nodes in S so that $S \cup B' \cup C$ becomes the desired CDS*.

In order to build the set C of connectors we adopt the following strategy: Initially, the set C is empty. Each node $b \in S$ finds shortest-distance paths to all neighbors $b' \in S$ ($b \neq b'$ and $id(b) > id(b')$) which are two and three hops away from b. If the length of a shortest path between b and b' is 2 and they are already connected via an intermediate node in B', then b does not choose any connector, else a connector is chosen. Ties are broken when there are more than one such connectors by selecting the one which has the smallest id. Similarly if they are three hops away and already connected by two (or one) nodes in B' then no (or one) connectors are sought. Otherwise, they select two connectors. All these connectors color themselves black. C denote the set of all such connectors.

In the following lemma, we prove that CDS$^* = S \cup B' \cup C$ in G forms a CDS.

Lemma 3.4.4 *The set* $CDS^* = S \cup B' \cup C$ *forms a CDS in* G.

Proof We first show that $S \cup B' \cup C$ is a dominating set in G and then prove that it is connected. By construction, $S \cup B'$ is a dominating set in G and so is $S \cup B' \cup C$.

Assume for the sake of contradiction that the subgraph G_s induced by the nodes in $S \cup B' \cup C$ is not connected. Consider two neighboring non-empty components G'_s and G''_s of G_s. There are two cases:

Case 1: There is no element of S in one of the components G'_s and G''_s. Without loss of generality assume G''_s is that component. Since G''_s is non-empty and does not have any element from S, the orphan nodes in G''_s must have selected some black nodes in B' to dominate them, otherwise G_s would not be a dominating set. If such a black node $u \in B', G''_s$ would not have a neighbor $v \in S, G'_s$, then u must belong to S, a contradiction.

Case 2: There is at least one node from S in each component $G's$ and G''_s. Find two nodes $s_1 \in S, G'_s$ and $s_2 \in S, G''_s$ such that the distance between them is at most three hops. Without loss of generality, let $id(s_1) > id(s_2)$. For each node $u \in S$ the algorithm selects at most two intermediate nodes to all other nodes in $v \in S$ which are at most 3 hops away and $id(u) > id(v)$. If s_1 and s_2 are not connected by any node in $B' \cup C$ then either their distance is more than three or their ids are the same, both cases yield a contradiction. ∎

We use the following lemma from [13] which bounds the size of C in G in terms of the cardinality of an MIS.

Theorem 3.4.5 *The cardinality of* C *is at most* $18 * |S|$ *[13].*

Theorem 3.4.6 *For any maximal independent set S in B, a set B' of additional nodes and a set C of connectors in a UDG $G = (V, E)$, we have $|S \cup B' \cup C| \leq 50.54 * |MCDS| + 38$.*

Proof Combining the results of Theorems 3.4.3 and 3.4.5 we get:

$$\begin{aligned} |C| &\leq 18 * |S| \\ &\leq 18 * |S \cup B'| \\ &\leq 18 * 2.66 * |MCDS| + 36 \\ &= 47.88 * |MCDS| + 36. \end{aligned}$$

Then,

$$\begin{aligned} |S \cup B' \cup C| &\leq |S \cup B'| + |C| \\ &\leq 2.66 * |MCDS| + 2 + 47.88 * |MCDS| + 36 \\ &= 50.54 * |MCDS| + 38. \end{aligned}$$

∎

One feature of our algorithm A is that it guarantees an MCDS (minimum CDS) for trees. The interesting thing is that we do not need to check whether G is a tree to compute an MCDS. This means that if G is a tree, algorithm A generates the optimal CDS. Other algorithms such as [3] require G to be checked for being a tree in order to construct a CDS. That is, the problem with their algorithm is that the nodes need to construct a spanning tree from the underlying graph which is an expensive operation in terms of energy usage.

We prove the optimality result in the following theorem.

Theorem 3.4.7 *If the underlying unit disk graph G is a tree with $|V| > 2$ algorithm A produces an MCDS.*

Proof It is well known that for a tree an MCDS consists only of the internal nodes of the tree. We prove the theorem by contradiction. Suppose a CDS produced by algorithm A consists of some leaves of the tree. Let ℓ be such a leaf and p its parent. Since ℓ is in the CDS, its computed convex hull $CH(N[\ell])$ is not included in any of its neighbor's convex hull and its color is black. However, its only neighbor is p and p's neighborhood contains at least two neighbors including ℓ, otherwise p is a leaf. Obviously, the convex hull $CH(N[\ell])$ is included in $CH(N[p])$, since $N[\ell]$ includes only ℓ and p. So ℓ would be colored grey and cannot be in the CDS, which is a contradiction. ∎

We analyze the performance of our algorithm in grids where we show that the cardinality of the CDS produced by algorithm A is less than twice the size of an MCDS.

Theorem 3.4.8 *Given a $m \times n$ grid, where nodes are placed at the intersection points and the Euclidean distance between two adjacent nodes equals 1, algorithm A produces a CDS such that $|CDS^*| \leq 2 * |MCDS|$.*

Proof By analyzing the $m \times n$ grid, the size of the minimum connected dominating set can be calculated as: $|\text{MCDS}| = \frac{m-1}{2} * n + \frac{m-1}{2} - 1 = \frac{mn-n+m-3}{2}$, when m is odd and $|\text{MCDS}| = \frac{m}{2} * n + \frac{m}{2} - 1 = \frac{mn+m-2}{2}$, when m is even. The cardinality of the CDS generated by algorithm A is mn. Thus if m is even the approximation factor is $2 \times \frac{mn}{mn+m-2} < 2$. However, if m is odd and $m > n$, the approximation factor is $2 \times \frac{mn}{mn-n+m-3} < 2$. ∎

Corollary 3.4.9 *For any induced subset of an $m \times n$ grid, algorithm A produces a CDS with at most twice as many nodes as an MCDS.*

3.4.1 Time and Message Complexities

According to our model $\mathcal{LOCAL}_S$, the time complexity of algorithm A is derived as follows. Each node u receives the location information of its neighbors and sends its location to them which can be done in a single round. Then u computes the convex hull $CH(N[u])$ and checks whether the convex hull is included in any of the convex hulls of its neighbors. However, the whole process of computing and comparing $CH(N[u])$ with $CH(N[v])$, $\forall v \in N(u)$, for inclusion is negligible in synchronous systems [61], where the time complexity is measured in terms of number of rounds. After computing $CH(N[u])$, u sends the list of vertices of $CH(N[u])$ to all its neighbors and receives theirs' as well. Since the message size is arbitrary, the exchange of convex hulls can be done in a single round. The computation of the distributed MIS [66] takes $O(\log^* |B|)$ time, where B is the set of black nodes. The distributed MIS algorithm in B generates the set S. Each node in S finds at most two intermediate nodes in S which are at most three hops away from it. However, the number of such neighbors is a constant [72] for a node $u \in S$. Thus computing such shortest paths takes O(1)-time. Therefore, the overall time complexity of the algorithm is $O(\log^* |B|)$=$O(\log^* n)$.

To compute the message complexity, we see that each node has to send its location, id (all these can be packed in a single message) to its neighbors, which takes $O(n)$ messages to be sent. Again, after computing the convex hull $CH(N[u])$, node u sends $CH(N[u])$ (the nodes in the convex hull are put in a message) to its neighbors which

also takes $O(n)$ messages to be exchanged in the network. However, to compute an MIS it takes $O(|E| * \log^* n)$ [66] messages. Thus the message complexity of the entire algorithm is $O(|E| * \log^* n)$.

3.5 Conclusion

Computing a CDS distributedly in WSNs is a very well studied problem. However, devising distributed algorithms for constructing such CDSs become more challenging when it is aimed to obtain a small constant approximation factor with each node knowing a limited amount of information about the topology of the underlying network. Several distributed algorithms are known in this context and interestingly almost all of them use the same approach: find an MIS in the network graph and then connect them through other nodes to construct the set.

Instead of finding an MIS in the whole network, our approach first builds a connected subgraph in the network graph, then finds an MIS in the subgraph and finally connects the nodes in the MIS through other nodes. By this technique, we could remove a number of unwanted nodes from being in the connected dominating set. This way our simple distributed algorithm guarantees a relatively small constant approximation factor which is a good improvement over the best known constant factor [13]. The main difficulty in lowering the constant factor lies in the reduction of the size of the connectors used to connect the nodes in the MIS. Exploring further geometric properties of unit disk graphs may help achieve a better bound for the size of the set of connector nodes. In future work, we look forward to finding distributed constant factor CDS algorithms in special classes of graphs, such as for example, planar graphs.

Chapter 4

Family of Connected Dominating Sets

In the last chapter, we studied the problem of constructing a single CDS in WSNs. However finding a CDS (even the MCDS) and using it all the time can exhaust the energy of the nodes of the CDS faster than other nodes in the network. Therefore, devising strategies to obtain a number of different CDSs and employing them one by one is a challenging task in the context of WSNs.

In this chapter, we investigate the problem of computing a family of connected dominating sets in a WSN in a distributed manner. We present a distributed algorithm that computes a family $S_1, S_2, \cdots, S_m$ of non-trivial CDSs with the goal to maximize $\alpha = m/k$, where k=$\max_{u \in V} |\{i : u \in S_i\}|$. In other words, we wish to find as many CDSs as possible while minimizing the number of frequencies of each node in these sets. Since CDSs play an important role for maximizing network lifetime when they are used as backbones for broadcasting messages, maximizing α reduces

the possibility of repeatedly using the same subset of nodes as backbones. We provide an upper bound on the value of α via a 'nice' relationship between all minimum vertex-cuts and CDSs in the network graph and present a distributed algorithm for the α maximization problem. For a subclass of UDGs, it is shown that our algorithm achieves a constant approximation factor of the optimal solution. As before a WSN is modelled as a UDG.

4.1 A Motivating Example

We are motivated by an important observation for the general CDS construction in WSNs where the main goal is to obtain a "small" approximation factor for the computed CDS, since finding a minimum cardinality CDS is NP-hard [16]. Consider the wheel graph in Figure 4.1 (a) with 9 nodes. Assume that each node has 9 units of energy and each transmission for a unit of data from a node to its neighbor consumes one unit of energy, neglecting the consumption of energy for message reception. The optimal CDS contains only node 9 at the center.

Suppose each border node $\{1, 2, \cdots, 8\}$ has a unit of data to be broadcast via the center node 9 in each communication round. That is, whenever a node on the periphery sends a message, it is node 9 which broadcasts (relays) it and all other nodes receive the message. After eight rounds corresponding to eight border nodes transmitting their data through the center node, the center node will be left with 1 unit of energy while all the border nodes still contain eight units of energy each. After the 9th round, node 9 will have completely exhausted its energy and the other nodes will have 8 units of energy each for continuing operation. Thus finding an optimal solution might not always be helpful in scenarios like this and it is not hard

to construct a family of such examples.

One easy way to get around this problem is to devise algorithms that can in some way evenly distribute the energy consumption among the nodes in the network by not using the same subset of nodes for all broadcasting operations. Clearly this may not yield a 'small-sized' or constant factor approximation solution to the CDS problem but can be used to prolong the lifetime of the network which depends on the functioning of every node in the network. Using node 9 as a CDS for one broadcasting operation then leaving it free and engaging six consecutive nodes from the border nodes as next CDSs for successive rounds (See Figure 4.1(b), (c)) for each broadcasting clearly enhances the overall network lifetime. In other words, we have two scenarios. First, we can use only the middle node as the CDS which gives $m/k = 1$, where m is the number of CDSs generated and k is the maximum number of occurrences of a node in these CDSs. In the second scenario, we can use 3 CDSs, where the nodes are taken from the border nodes only and one CDS containing only the middle node. Thus we get $m/k = 4/3$ which means that if the network is used for 4 rounds, for the first scenario, the middle node is used for 4 rounds, and in the second scenario, no node is used for more than 3 rounds.

In general, the above problem may be solved by finding a *large* number of *disjoint connected dominating sets* since doing so has a direct impact on conserving the energy of individual sensors and hence prolonging the network lifetime. However, as we will show, it is not always possible to find a large number of such disjoint sets because the number of disjoint CDSs is bounded by the size of a minimum vertex-cut of the graph. Thus, the main focus is to find as many CDSs as possible while minimizing the frequency of participations of each node in the network. In other words, we would like

to maximize $\alpha = m/k$ which would increase the overall network lifetime by incurring less energy consumption on the part of each node for broadcasting.

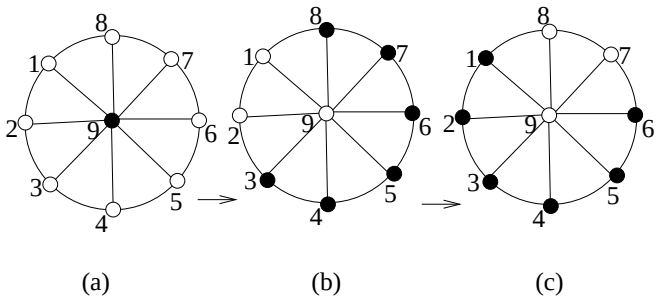


Figure 4.1: Family of CDSs.

We provide a novel local algorithm (that requires only the knowledge of nodes at most 3-hops away) to compute a family of CDSs with a goal to maximize α. Our approach for obtaining such a family of sets depends on the computation of maximum cliques using only the one-hop neighborhood of each node combined with a simple coloring scheme for the nodes that requires each node to know information of its neighbors at most 3-hops away. Maximum cliques in UDGs can be found in polynomial time [16, 30].

The rest of the chapter is organized as follows. In Section 4.2 we describe related work. Section 4.3 presents definitions and assumptions that are used throughout the chapter and formulates the problem as well. The distributed algorithm is presented in Section 4.4. A theoretical analysis of our algorithm is presented in Section 4.5 followed by conclusion in Section 4.6.

4.2 Related Work

There is a large body of literature [2, 3, 15, 23, 28, 56, 70, 47] regarding finding an MCDS in WSNs which focuses on the construction of a *single* CDS and the effort is to devise algorithms to produce a small approximation factor for it. Aside from looking for a single CDS with a small approximation factor, there has been some research towards finding a family of dominating sets [58, 62, 21, 69] for data gathering and sensing operations in sensor networks. In [21], the authors consider the problem of finding a maximum number of disjoint dominating sets called the *domatic partition* (we study this problem in the next chapter). They show that every graph with maximum degree $\triangle$ and minimum degree δ contains a domatic partition of size $(1 - o(1))(\delta + 1)/\ln \triangle$. They provide a centralized algorithm to produce a domatic partition of size $\Omega(\delta/\ln\triangle)$.

The papers [62, 58, 69] are somewhat related to ours since they investigate the problem of maximizing the lifetime of wireless networks by generating disjoint dominating sets (not connected dominating sets) for data gathering purposes. The general focus is that the nodes in the dominating set act as "coordinators" for the network and the rest of the nodes remain in an energy-efficient *sleep* mode. Thus to maximize the lifetime of the network, it is important to rotate the coordinatorship among the nodes in the network so that every node is roughly given an equal opportunity to go into sleep mode.

In particular, the authors in [69] propose a centralized algorithm, without providing any worst case analysis or bound, to generate a number of disjoint dominating sets in order to cover a geometrical region by network nodes for as long as possible. A general version of the domatic partition called k-domatic partition ($k \geq 2$), where

in each dominating set a node is dominated by at least k nodes, is considered in [62]. Their algorithms construct a $k-$ domatic partition of size at least a constant fraction of the largest possible $(k-1)$-domatic partition.

In [58], the authors look at a variant of the above problem and propose a randomized algorithm for maximizing the network lifetime. The problem considered in [58] is called the Maximum Cluster-Lifetime problem, where they aim to find an optimal schedule consisting of a set of pairs $(D_1, t_1), \cdots, (D_k, t_k)$, where D_i is a dominating set and t_i is the time during which D_i is active. The problem then asks for the schedule S with maximum length $L(S)$ such that $\sum_{i:v \in D_i} t_i \leq b_v$, where b_v is the maximum time that v can be in a dominating set. The randomized algorithm computes a schedule which is an $O(\log n)$-approximation algorithm with high probability.

4.3 Preliminaries and Problem Formulation

Each node u has an ordered triple, $(ct(u), deg(u), id(u))$, where the first element $ct(u)$ denotes the number of times u has already participated in previous CDSs, $deg(u)$ and $id(u)$ denote the degree and id of u. Initially, $ct(u) = 0$, $\forall u \in V$, and this is updated as the algorithm is executed. For a maximum clique $X \subseteq N[u]$ and $u, v \in X$, we say node u is lexicographically smaller than node v if $(ct(u), deg(u), id(u)) < (ct(v), deg(v), id(v))$, i.e., either $ct(u) < ct(v)$ or $ct(u) = ct(v)$ and $deg(u) < deg(v)$ or $ct(u) = ct(v)$ and $deg(u) = deg(v)$ and $id(u) < id(v)$. The rank of node u with respect to clique X denoted by $r(X(u))$ is the index of the lexicographically sorted (ascending order) triples of the nodes of X. Note that the rank $r(X(u))$ is changed by the update of the variable $ct(u)$.

4.3.1 Problem Formulation

Given a UDG $G = (V, E)$, we would like to find a family of subsets $S_1, S_2, \cdots, S_m$ such that

i) $\forall i$, S_i is a CDS

ii) and $\alpha = m/k$ is maximized, where k=$\max_{u \in V} |\{i : u \in S_i\}|$.

4.4 A 3-local CDS Algorithm

We provide a local algorithm for generating a family of CDSs. The algorithm uses a simple coloring scheme where initially all nodes are white and when a node becomes a member of the CDS S_i, it becomes black. Set S_i, whose members are all black, becomes the virtual backbone and remains active for a *time-slot* (consisting of fixed units of time) to perform broadcasting. After the time-slot, all black nodes change their color into white. Then the algorithm (part of it) is executed again and a new set S_{i+1} of black nodes is formed which remains active for another time-slot and so on. In this section, we present a simple algorithm for computing a family of CDSs in G which is based on coloring the nodes of G. This algorithm appears in [39]. Here we outline the algorithm which consists of two phases described in what follows.

4.4.1 Dominating Set Generation: Phase I

In Phase I, each node u computes a maximum clique in $N[u]$ following the polynomial time algorithm [30] which computes maximum cliques in UDGs. Let $C(u)$ denote the maximum clique computed by u. Henceforth maximum cliques are just called cliques unless otherwise stated. Node u sends the nodes in $C(u)$ to all its direct neighbors v

if $v \in C(u)$. Similarly u receives a clique $C(v)$ from $v \in N(u)$ if $u \in C(v)$. Denote the set of all such cliques received by u as $C(N(u))$, i.e., $C(N(u)) = \bigcup_{u \in C(v), v \in N(u)} C(v)$. For each clique $C^+ \in (C(u) \cup C(N(u)))$, u computes its rank $r(C^+(u))$ which gives u, $|C(u) \cup C(N(u))|$ ranks in total. For each C^+, the smallest ranked node of C^+ becomes active by coloring itself black. This yields a set of black nodes B in G. We prove in the next section that B forms a dominating set. Now we need to add a subset C' of nodes (call them connector nodes) from the remaining white nodes and color them black to make $B \cup C'$ a CDS. In Phase II, we determine such set C'.

This phase of the algorithm which is executed at each node u, is given in Figure 4.2.

PHASE I:

1: Broadcast $id(u)$, $deg(u)$ and location to $v \in N(u)$
2: For each $v \in N(u)$, receive $id(v)$, $deg(v)$ and v's location
3: Compute $C(u)$ and send $C(u)$ to $v \in C(u), v \in N(u)$
4: Receive $C(v)$, form $C(N(u)) = \bigcup_{u \in C(v), v \in N(u)} C(v)$
5: For each $C^+ \in (C(u) \cup C(N(u)))$, compute $r(C^+(u))$
6: Color u black if $r(C^+(u)) < r(C^+(v))$, $\forall v \in C^+ \setminus \{u\}$, $\exists C^+ \in (C(u) \cup C(N(u)))$

Figure 4.2: Phase I of the algorithm.

4.4.2 Connected Dominating Set Generation: Phase II

Recall that Phase I finds a dominating set B (consisting of black nodes) in G. In Phase II we compute a connected dominating set, that is, we select a set of connectors C' to connect the nodes in B such that $B \cup C'$ becomes a CDS. Since the distance between any two closest nodes in B can be at most three, we divide this phase into two cases, where in Case 1 we discuss how two nodes in B which are two hops away

from each other connect themselves via connectors. Case 2 is the same but such nodes are exactly three hops away from each other.

Case 1: Consider a node $u \in B$. Node u selects a connector node for each node $v \in B$, exactly two hops away from it and $id(u) > id(v)$, if there does not exist any black node $w \in N(u) \cap N(v)$. If there are multiple candidates for such a connector, the lexicographically smallest one is chosen. Node u send a 'YES' message to the selected connector (denote the connector node c) to turn itself into black.

Case 2: In this case u selects at most two connectors to connect a node $v \in B$ which are exactly three hops away from it and $id(u) > id(v)$. If there already exists two intermediate black nodes (belonging to B) between u and v, then u does nothing. Otherwise, the three-hop shortest paths between u and v must contain either one black node or no black nodes. In the first situation the lexicographically smallest black node is chosen among all such black nodes. After selecting the black node a white connector c is selected. Similarly, if there is no black node between u and v, then u selects two connectors c', c'' such that the average $ct(.)$ values of c' and c'' (that is, $(ct(c') + ct(c''))/2)$ is smallest (ties are broken arbitrarily). Node u then sends a 'YES' message to all such connectors (c or c' and c'') it thus selects. After receiving a 'YES' message, connectors turn themselves into black.

Thus the executions of Phases I and II form a CDS S_i which consists of black nodes and these nodes remain active for a certain amount of time (a time-slot) for broadcasting while the white nodes stay in the energy-efficient sleep mode. Phase II is given in Figure 4.3. After serving for a time-slot, all nodes in the CDS, i.e., black nodes, update their $ct(.)$ values by one, send the updated values to their neighbors and change color into white. All nodes in G then start executing from Line 5 Phase I and

PHASE II:
1: **If** u is black **Then**
2: **If** $v \in B$ and v is two-hop away from u and $id(u) > id(v)$
3: Find at most a connector c //(lexicographically smallest)
4: Send a 'YES' message to c //(if there exists such a c)
5: **Endif**
6: **If** $v \in B$ and v 3-hop away from u and $id(u) > id(v)$ **Then**
7: Find at most two connectors (c' or c'')
8: Send a 'YES' message to c' and c'' //(if there exist such connectors)
9: **Endif**
10: **Endif**
11: **If** u is white and receives a 'YES' message **Then**
12: Color u black //(u is a connector)
13: **Endif**
14: **If** u is black **Then**
15: Remain active for a round (black nodes form the current CDS)
16: Send $ct(u) = ct(u) + 1$ to $N(u)$
17: Turn u white //(Just before the next round, turn every node white)
18: **Endif**

Figure 4.3: Phase II of the algorithm.

then Phase II with their updated triples. This generates a new CDS S_{i+1} consisting of black nodes, which remains active for another time-slot. Then $ct(.)$ values are updated and a new CDS S_{i+2} is constructed, and so on.

Figure 4.4 illustrates the construction of a CDS by coloring certain nodes black. The numbering of nodes denotes the id of the corresponding nodes and the number inside the bracket indicates the nodes' $ct(.)$ values. Initially all $ct(.)$ values are set to zero and they are incremented by one each time the respective nodes participate in the CDS. In this example we can construct six different CDSs.

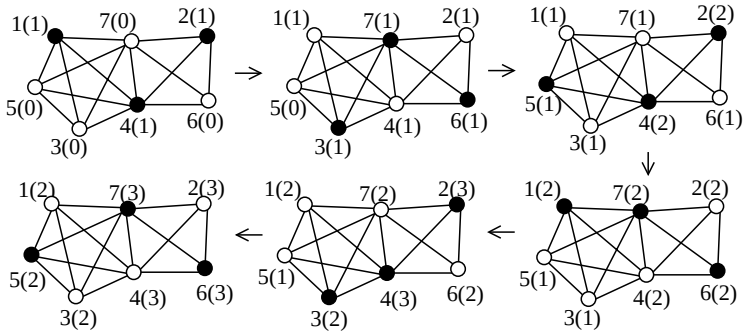


Figure 4.4: Illustration of the algorithm to produce a family of CDSs.

4.5 Theoretical Analysis

In this section we give an overview of the theoretical analysis of our algorithm. First we show that Phase I computes a dominating set and then we prove that Phase II forms a CDS. We analyze some other characteristics of our algorithm and provide a relationship between the CDSs and minimum vertex-cuts of G.

Lemma 4.5.1 *The set of black nodes B produced in Phase I forms a dominating set in G.*

Proof We prove the claim by contradiction. Consider a node u. If it is black or connected to a black node then we are done. Otherwise, all nodes in $N[u]$ are white. Let the clique computed by u be $C(u) \subseteq N[u]$. Since each node in $C(u)$ sorts the ordered triples lexicographically, each of them has a unique rank from $\{1, \cdots, |C(u)|\}$. The node with the least rank in $C(u)$ must be black, a contradiction. ∎

Lemma 4.5.2 *The set of black nodes $B \cup C'$ produced in Phase II forms a CDS in G.*

Proof Assume the subgraph G' induced by $B \cup C'$ is not connected. Since G' is not connected, it must have at least two components. Without loss of generality, assume G'_1 and G'_2 are two components of G' and each contains at least a node from B. Let $u \in G'_1$ and $v \in G'_2$ be two nodes such that the shortest distance between them is at most three (this is possible since B is a dominating set in the underlying graph G). Since either $id(u) > id(v)$ or $id(v) > id(u)$, the node with the larger id will find at most two connectors to connect the node with the smaller id. If they do not find such connectors then either the shortest distance between them is not at most three or they are already connected by some connectors. In both cases this is a contradiction. ∎

4.5.1 Time and Message Complexities

Each node requires only the information of its three-hop neighborhood. Following the model $\mathcal{LOCAL}_S$, collecting information from a three-hop neighborhood can be done in constant time. Thus the time complexity of the entire algorithm is constant since local computation time (the time spent to compute the cliques in each node's neighborhood) is negligible in a synchronous system [61].

For the message complexity analysis, we see that in Phase I each node sends a single message (containing its id, position) to its neighbors, thus generating O(n) messages in total in the network. After computing a clique in its 1-hop neighborhood, node u sends this information (e.g., ids of the nodes in the clique) in a single message to its neighbors and similarly receives cliques from its neighbors. This again makes a total of O(n) messages.

For Lines 1-5 in Phase II, each node sends a single message, and also relays at

most 1 message (that is, it receives all messages from its neighbors and merges them into a single message and sends the message) totaling O(n) messages. For the rest of the algorithm in Phase II, that is, in Lines 6-18, there are O(n) messages generated (a node relays at most two messages and generates its own message). These many messages are required to explore black nodes which are 3-hops away from a certain node. This complexity arises for one single round. Since there can be at most $\triangle$ rounds, the message complexity becomes O($n\triangle$). Thus the message complexity of the entire algorithm is O($n\triangle$).

4.5.2 Relation between Minimum Vertex-cuts and CDSs

We prove that the maximum value of m is upper bounded by a number that results from a relation between minimum vertex-cuts and any CDS (including an MCDS) in a graph. A *vertex-cut* of G is a set of vertices whose removal renders G disconnected. A vertex-cut of minimum cardinality is called a *minimum vertex-cut*, $\kappa(G)$. Let $\mathcal{K}(G)$ denote the family of all minimum vertex-cuts of G. We show that for any $\kappa(G) \in \mathcal{K}(G)$, $\exists a \in \kappa(G)$ such that $a \in S_i$, where S_i denotes any CDS of G. In other words, any CDS must take at least one element from each of the minimum vertex-cuts. Instead of 'vertex-cut' we use 'node-cut' in order to be consistent with the terminology of the discussion.

Theorem 4.5.3 *In a connected graph G, any CDS (including an MCDS) must contain at least one node from each $\kappa(G)$, where $\kappa(G) \in \mathcal{K}(G)$.*

Proof Consider any minimum node-cut $\kappa(G) \in \mathcal{K}(G)$ and suppose for the sake of contradiction that an arbitrary CDS S^* of G (recall that S^* can be an MCDS) does

not contain any node from $\kappa(G)$. Let G_1 and G_2 be two connected induced subgraphs generated by the removal of $\kappa(G)$. Set S^* is a CDS and does not contain any node from $\kappa(G)$, so components G_1 and G_2 must each have its own CDS, name them $S^*(G_1) \subseteq S^*$ and $S^*(G_2) \subseteq S^*$, respectively. However, $S^*(G_1) \cup S^*(G_2)$ is not connected since the only nodes that could connect $S^*(G_1)$ and $S^*(G_2)$ belong to $\kappa(G)$, thus contradicting the fact that S^* is a CDS. ∎

Here we show a relationship between the number of minimum node-cuts and the cardinality of an MCDS. The following corollaries are directly deducible from the above theorem.

Corollary 4.5.4 *In G, $|MCDS| \geq |\mathcal{K}(G)|$ if and only if $\kappa_i(G) \cap \kappa_j(G) = \phi$ for all i, j, $1 \leq i, j \leq |\mathcal{K}(G)|$.* ∎

Corollary 4.5.5 *If G is a tree, then every internal node is a minimum node-cut. Therefore, $|MCDS| = |\mathcal{K}(G)|$=number of internal nodes in G.* ∎

Now we have the following lemma.

Lemma 4.5.6 *The optimal value of α, α_{opt} is bounded by the size of a minimum node-cut in $\mathcal{K}(G)$.*

Proof Let M be the set of CDSs the optimal algorithm finds in G and $\kappa(G)$ be a minimum node-cut in $\mathcal{K}(G)$. Recall that all $\kappa(G)$s in $\mathcal{K}(G)$ are of the same size, otherwise they would not be minimum node-cuts. Since every CDS in M must use at least one node from $\kappa(G)$ (Theorem 4.5.3), the maximum value of k (i.e., the maximum number of occurrences of a node in M) will be at least $|M|/|\kappa(G)|$. Hence, $\alpha_{opt} \leq |M|/(|M|/|\kappa(G)|) = |\kappa(G)|$ and the claim follows. ∎

Now we analyze the performance of our algorithm towards optimizing the value of $\alpha = m/k$ for a small subclass of UDGs. We consider a particular subclass of UDGs, where each node $u \in V$ belongs to exactly one clique (recall clique means maximum clique). That is, each node computes a clique $C(u)$ but does not receive any clique $C(u') \neq C(u)$ from $u' \in N(u)$ (in other words, $|C(u) \cup C(N(u))| = 1$). There are also two constants c and d. For any node u, assume its neighbors belong to at most c different cliques and for d we define the following. Let the set of neighbors of u which belong to cliques $C(u') \neq C(u)$ be $N'(u)$. That is, $N'(u) = \{u''|u'' \in N(u), u'' \in C(u'), C(u') \neq C(u)\}$. Let the number of such neighbors of u be bounded by d, that is, $|N'(u)| \leq d$. Denote by α_{alg} the value of α obtained by our algorithm.

Lemma 4.5.7 *For the subclass of UDGs, defined in the previous paragraph,* $\alpha_{alg} \geq 1/(c * (d+1)) * \alpha_{opt}$.

Proof Let there be s different cliques computed by the nodes in V and $p(> 2)$ denote the cardinality of the smallest-sized clique among these cliques. Note that the maximum number of disjoint CDSs that are possible in this setting, can be at most p, implying that the optimal value α_{opt} can be also at most p. This is because generating the $(p+1)$-th CDS must reuse some of the nodes already used in previous CDSs, making $k > 1$ (recall that k is the maximum number of occurrences of a node in the generated CDSs). Thus $\alpha_{opt} \leq p$.

In order to find a CDS, our algorithm first selects a node from each of the s cliques in Phase I. Let u be the node selected from clique $C(u)$. Since u's neighbors can be at most in c different cliques, u requires at most $c-1$ *additional* nodes from $C(u)$ that act as connectors to connect to $c-1$ other cliques. Thus the algorithm uses at most c nodes from $C(u)$ for each CDS (see Figure 4.5 for such an illustration).

Therefore, our algorithm can compute at least p/c CDSs. Now we determine the value of k, that is, the upper bound in the number of participations of a node in different CDSs. Since $|N'(u)| \leq d$, u can be selected at most $d+1$ times, once it is selected in Phase I, other times (at most d times) it is selected (in Phase II) as a connector between two nodes in different cliques. Hence $k \leq (d+1)$. Therefore, $\alpha_{alg} \geq p/1/(c*(d+1)) \geq 1/(c*(d+1)) * \alpha_{opt}$. ∎

An example of this subclass is given in Figure 4.5 with $c = 3$ and $d = 2$.

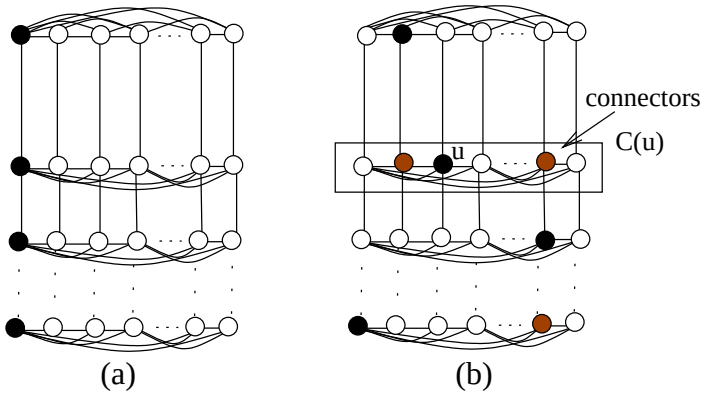


Figure 4.5: Illustration of Lemma 4.5.7. (a) An optimal algorithm chooses any column of nodes as a CDS each time (shown in black), whereas our algorithm (b) first chooses black nodes from each clique and then uses at most two *additional* (shown in brown) nodes from each clique to make them connectors between cliques.

Theorem 4.5.8 *For a complete graph K_n, our algorithm achieves $\alpha_{alg} = \alpha_{opt} = n$.*

Proof For graph $G = K_n$, initially in Phase I, the algorithm selects the node with the lowest id by coloring it black since all nodes have the same $ct(.)$ value and degree. This node updates its $ct(.)$ value by one. In Phase II, the selected black node does not find any other black node in its two or three-hop neighborhood which means that

no connector node is required. Thus we have exactly one node as CDS, i.e., $|S_1| = 1$. In the next round, the second lowest id node is selected and used as the only node in the CDS ($|S_2| = 1$), and so on. Thus we have a family $S_1, S_2, \cdots, S_m$ of CDSs, where each set S_i consists exactly of one node of K_n, all nodes in K_n are used in CDSs and $S_i \neq S_j$, $i, j \in [1, m]$. This means $m = n$ and $k = 1$. Therefore, $\alpha_{alg} = \alpha_{opt} = n$. ∎

4.5.3 Generating Disjoint CDSs: A counterexample

It seems reasonable that generating all possible disjoint CDSs for the problem of maximizing α can obtain the maximum value of α. This is because the maximum number of frequencies of a node in these sets will be guaranteed to be at most 1 and maximizing the number of disjoint CDSs will maximize the value of α. However, in this section we provide a counterexample showing that finding all possible disjoint CDSs does not necessarily maximize α. Consider the geometric graph shown in Figure 4.6 (a). In this case, the number of possible disjoint CDSs is 1 (shown in Figure 4.6 (b)) for which the value of α is, $\alpha = m/k = 1$. On the other hand, the value of α can be maximized by finding overlapping CDSs as shown in Figure 4.6 (c)), where we have 3 CDSs and the maximum number of participations of a node is 2 resulting in $\alpha = 3/2 = 1.5$.

4.6 Conclusion

In this chapter, we have presented a local algorithm for generating a family of CDSs in sensor networks with a view to increasing the lifetime of nodes in the network. It is shown that using the same CDS (even an optimal one) in sensor networks is not

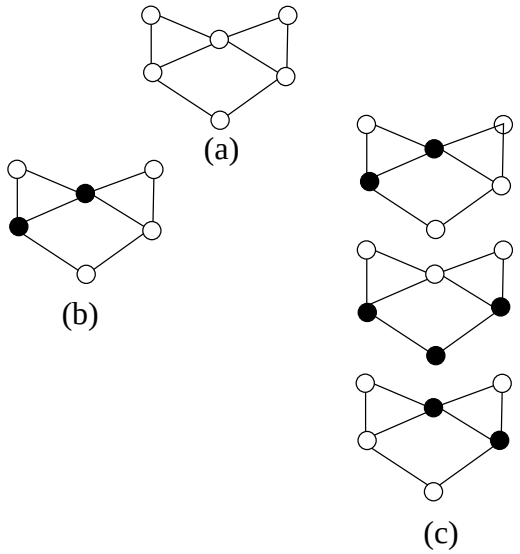


Figure 4.6: A counterexample showing that obtaining the maximum number of disjoint CDSs does not always maximize α.

a viable option since the nodes in the set quickly deplete their energy due to their participation in every broadcast. Inspired by this observation, we develop a local algorithm which takes care of this situation by generating other CDSs so that the burden of broadcasting can be distributed over the nodes. We show an interesting relationship between minimum vertex-cuts and CDSs, where the cardinality of a minimum vertex-cut limits the number of *disjoint* CDSs. We believe this is the first local algorithm of its kind in the context of WSNs since previous algorithms mainly focus on generating a single CDS of small size. Our algorithm obtains a constant factor approximation of the value of α for a small subclass of UDGs. However it remains as an interesting open problem to devise an algorithm (centralized or distributed) with a constant approximation factor for UDGs.

Chapter 5

Domatic Partition

Here we consider a variant of the problem studied in the previous chapter, in which we would like to generate as many disjoint dominating sets as possible, in other words, we wish to maximize the number of dominating sets where the maximum number of participations of a node in these sets is at most 1.

It is well known in WSNs research that distributing sensing and data gathering tasks to a dominating set (preferably small-sized) helps prolong the network lifetime by engaging such a subset of nodes for these tasks and letting other nodes go into the energy-efficient sleep mode. For these types of applications, it is generally assumed that the nodes in a dominating set need not form a connected subgraph since these nodes are expected to individually sense the environment, collect and process the data obtained and finally send them directly to the base station. Because they are busy all the time for sensing, processsing, and transmitting data, nodes in the dominating set quickly run out of energy. One possible way to overcome this situation is to find a number of different dominating sets in the network and use them one by one iteratively.

In this chapter, we investigate the problem of finding the maximum number of disjoint dominating sets called the domatic partition problem in UDGs. Although the problem is NP-hard in general graphs [26], it is unknown whether the same is true for UDGs. However, we present an algorithm towards solving this problem together with experimental results and give a conjecture based on our results about the maximum number of disjoint dominating sets in UDGs.

5.1 Introduction

Recall that a subset $D \subseteq V$ of $G = (V, E)$ is a dominating set if every vertex $v \in V \setminus D$ has at least a neighbor in D. Here, we consider the problem of generating subsets $D_1, D_2, \cdots, D_k$ such that each $D_i \subseteq V$ is a dominating set and $D_i \cap D_j = \phi$, $i \neq j$, and the objective is to maximize k. Dominating sets play an important role in saving energy in individual sensors in WSNs which will be clear from the following discussion. In general it is expected that the size of a dominating set be as small as possible.

Once deployed with a finite amount of energy, nodes in sensor networks continue to spend energy sensing, gathering, and distributing data from the environment. Since the nodes cannot be replenished with energy, one viable option is to prolong the lifetime of the network by carefully utilizing the energy of individual nodes for these tasks. In order to achieve this goal, one obvious way is to exploit the physical proximity of nodes. Since closely positioned sensors in the plane sense almost similar data, it is important to make active as few sensors as possible from such nearby nodes, and put the rest of the nodes in the sleep mode to conserve energy. Thus, by keeping only a few nodes active for gathering, processing, and transmitting data we save valuable energy for other nodes. This naturally leads us to utilize the concept

of a dominating set, where the nodes selected for carrying on the sensing tasks on behalf of their nearby sensors form the dominating set. The smaller the size of the dominating set the better is the energy saving since more sensors can be put in the energy-efficient sleep mode.

However, as mentioned in the previous chapter, this kind of solution has one major drawback in terms of balanced energy distribution among the sensors. This is because the sensors in the dominating set will quickly consume their energy by sensing and transmitting data while the other inactive sensors (that do not belong to the dominating set and are in the sleep mode) can save their energy. This will result in disproportionate energy consumption among the sensors which is not expected in many applications where network lifetime depends on the functioning of individual sensors. Even finding the minimum dominating set, that is, the one with the fewest number of nodes, will be of no help in this scenario.

This problem can be alleviated if we replace an existing dominating set D_i with another disjoint dominating set D_j in the network and place the nodes in D_i in the sleep mode. This facilitates maximizing the network lifetime since the active role is now shifted to a completely new set of nodes. Thus the basic idea is to find as many disjoint dominating sets as possible and use each one for a certain amount of time and then replace it with another one and so on. If initially all the nodes start with the same amount of energy, then one can expect that the energy consumption will likely be balanced among the nodes. The maximum number of disjoint dominating sets is bounded by the minimum degree of the graph plus one, since a node can either be in one of the dominating sets or be dominated by at least one of its neighbors. A graph which possesses the maximum number of disjoint dominating sets given by the

bound is called domatically full. An example of a domatically full graph is given in Figure 5.1(a). However, not all graphs are domatically full (see Figure 5.1(b)).

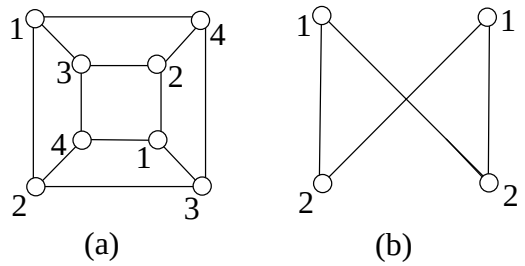


Figure 5.1: (a) In this 3-connected graph (it is a general graph, not a UDG), we have four disjoint dominating sets. Each dominating set is denoted by the same digit. Hence the domatic partition of this graph is 4. (b) The graph does not admit more than two disjoint dominating sets although the minimum degree plus one is 3.

If we can have a family $D_1, D_2, \cdots, D_k$ of disjoint dominating sets in $G = (V, E)$, where $D_i \subseteq V$, then we can employ the first dominating set D_1 for a certain time interval, say, $[0, t_1]$. Thereafter, we can replace D_1 by D_2 and make D_2 active for the $[t_1, 2 * t_1]$ time period, and so on until we are finished with D_k being active for $[(k-1) * t_1, k * t_1]$ time interval in which case we can repeat the whole process starting from D_1. It is clear that the larger the value of k, the longer we can prolong the lifetime of the network. However, as mentioned above, the optimal (maximum) value of k can be at most $\delta + 1$, where δ is the minimum degree in G. Here, we investigate the problem of finding the maximum number of disjoint dominating sets (called the domatic partition problem), that is, the maximum value of k (called the domatic partition number) in the special case of UDGs.

5.2 Related Work

The minimum dominating set problem is one of the classical graph theory problems that has been extensively studied [26, 41] and proven to be NP-hard in general. For general graphs, one can show [20] that unless $NP \subseteq DTIME\ (n^{O(\log\log n)})$, no polynomial time algorithm can approximate the minimum dominating set problem better than $\ln\triangle - o(\ln\triangle)$, where $\triangle$ is the maximum degree of the graph. In UDGs, the minimum dominating set problem still remains NP-hard [16]. However, it can be approximated within a constant factor. In [1], Alzoubi et al. show that the size of any MIS in a UDG is at most 4 times the size of the optimal dominating set. Since an MIS is also a dominating set, this proves that a dominating set is at most 4 times the size of the optimal dominating set. The dominating set problem has also been studied in distributed settings. In a distributed environment, a constant factor approximation ratio is obtained in [42] in $O(\log^2 n)$ time. Luby [55] gives a randomized distributed algorithm to find a constant approximation to the minimum dominating set problem in $O(\log n)$ time.

However, all the aforementioned algorithms focus on finding a single 'small' sized dominating set in the graph. As pointed out earlier, the domatic partition problem is one of the problems proved to be NP-hard in [26] for general graphs. This problem has been throughly studied by Feige *et. al.* [21] who provide some useful insights about the problem for general graphs. In [21], the authors prove that every graph with maximum degree $\triangle$ and minimum degree δ contains a domatic partition of size $(1 - o(1))(\delta + 1)/\ln\triangle$. They show that $1/(\ln\triangle)$ is the best possible approximation ratio for this problem for general graphs unless $NP \subseteq DTIME\ (n^{O(\log\log n)})$. A natural greedy approach to this problem is to find as many small-sized disjoint dominating

sets as possible. However, Fujita [22] studies several variants of such greedy algorithms and shows that their performance ratio is no better than $1/\sqrt{n}$.

In [58], the authors look at a variant of the domatic partition problem and propose randomized approximation algorithms for maximizing the network lifetime. The problem considered in the paper is called the Maximum Cluster-Lifetime problem, where they aim to find an optimal schedule consisting of a set of pairs $(D_1, t_1), \cdots, (D_k, t_k)$, where D_i is a dominating set and t_i is the time during which D_i is active. That is, D_1 is active in the time-interval (slot) $[0, t_1]$ and D_i is active in the time-interval $[\sum_{j=0}^{i-1} t_j, \sum_{j=0}^{i} t_j]$. The lifetime of a schedule is defined as $L(S) = \sum_{i=1}^{k} t_i$. The problem then asks for the schedule S with maximum length $L(S)$ such that $\sum_{i:v \in D_i} t_i \leq b_v$, where b_v is the maximum time node v can be in a dominating set. The randomized algorithms compute a schedule which is an $O(\log n)$ approximation to the Maximum Cluster-Lifetime problem with probability at least $1 - o(1/n)$.

The authors in [69] propose a centralized algorithm, without providing any worst case analysis or bound, to generate a number of disjoint dominating sets in order to cover a geometrical region by a network of nodes for as long as possible. A general version of the domatic partition called the k-domatic partition ($k \geq 2$), where in each dominating set a node is dominated by at least k nodes, is considered in [62]. The authors propose three deterministic distributed algorithms. Each of the algorithms constructs a $k-$ domatic partition of size at least a constant fraction of the largest possible $(k-1)$-domatic partition.

The reminder of this chapter is organized as follows. In Section 5.3 we provide definitions and assumptions that are used throughout the chapter. The proposed

algorithm is presented in Section 5.4. We provide a theoretical analysis of our algorithm in Section 5.5 followed by experimental results in Section 5.6. We conclude in Section 5.7.

5.3 Preliminaries and Definitions

Our algorithm to find disjoint dominating sets operates in time-slots (we use time-slots as iterations interchangeably) where at each time-slot i we construct a dominating set D_i (where $D_i \cap D_j = \phi, i \neq j$ and $i, j \geq 1$). We color the nodes in D_i with color class i. By $\delta = min_{v \in V} deg(v)$, we mean the minimum degree in the graph.

5.3.1 Compatible and Incompatible nodes

A node $v \in N[u]$ is called an uncovered neighbor of u at iteration i if there is no node $w \in D_i$ that is a neighbor of v (we say that v is not *covered* by any node in D_i). The total number of uncovered neighbors of u (including u) in iteration i is denoted as, $uncov_i(u) = |\{v | v \in N[u], vw \notin E, w \in D_i \wedge v \neq w\}|$. The *dom* number of node u, denoted by $dom(u)$, is the number of neighbors (including u) that belong to some dominating sets, that is, $dom(u) = |\{v | v \in N[u], v \in D_j, j \geq 1\}|$. Each node u maintains a triple at iteration i, $t_i(u) = \prec 1/uncov_i(u), dom(u), id \succ$ which is updated in each iteration. If $uncov_i(u) = 0$, then we assign $uncov_i(u) = 1/(n+1)$, where $n = |V|$. We call a node u *compatible* if $uncov_i(u) \neq 1/(n+1)$, otherwise it is *incompatible*. For any two compatible nodes u, v, we say u is smaller than v in iteration i if $t_i(u)$ is lexicographically smaller than $t_i(v)$ and denote the order by $t_i(u) < t_i(v)$. Note that comparison is done only between compatible nodes.

5.4 Algorithm for the Domatic Partition Problem

In this section we informally describe our algorithm. The algorithm works in iterations starting from iteration 1 and in each iteration i ($i \geq 1$), we compute a dominating set D_i such that $D_i \cap D_j = \phi, i \neq j, i, j \geq 1$. The algorithm runs for at most $\delta + 1$ iterations. Specifically, it stops in iteration j if we cannot form a new disjoint dominating set D_j or we have $j = \delta+1$. If $j = \delta+1$ then we have reached the optimal (maximum) number of disjoint dominating sets. Otherwise, $j - 1$ is the maximum number of disjoint dominating sets $D_1, D_2, \cdots, D_{j-1}$ returned by the algorithm. This algorithm has appeared in [38].

Begin with iteration 1. Let $D'_1 = \phi$ be a null set that expands gradually as nodes are inserted into it during the execution of the algorithm. First, we compute a triple for each node in G and select the smallest one among the compatible nodes. From the triples it is easy to see that we always find such a single node. Let u be the smallest node. We color u 1, add u to D'_1 (note that D'_1 is a temporary set that grows as dominating nodes are added to it). We update $t_1(v)$, $v \in N[u]$. Then we pick the smallest compatible node in $V \setminus D'_1$, color it 1, insert into D'_1 and update the nodes' triples. At each selection of a node u in D'_1, we make at least one node incompatible. We repeat the process until all nodes become incompatible. Assign $D_1 = D'_1$. We claim that nodes in D_1 form a dominating set. At this stage, $\forall_{u \in V} uncov_i(u) = 1/(n+1)$ and $\forall_{v \in V \setminus D_1} dom(v) \geq 1$.

We now describe in general how a dominating set D_i is formed at iteration $i \geq 2$. First, initialize $\forall_{u \in V} uncov_i(u) = deg(u) + 1$ and let $D'_i = \phi$. (As before D'_i is a temporary set used in the i-th iteration to contain the dominating nodes that are added to it as the algorithm progresses and when D'_i becomes a dominating set we

rename it as D_i). Select the smallest node among the compatible nodes in $V \setminus (D_1 \cup D_2 \cdots D_{i-1} \cup D'_i)$, color it i, and add it to D'_i. Update the triples of the corresponding nodes. Then find the smallest compatible node in $V \setminus (D_1 \cup D_2 \cdots D_{i-1} \cup D'_i)$, color it i and insert it into D'_i. Update the triples of the nodes whose $uncov_i(.)$, $dom(.)$ values are changed. Continue this process until one of following cases occurs

Case I: all nodes become incompatible

or

Case II: $V \setminus (D_1 \cup D_2 \cdots D_{i-1} \cup D'_i) = \phi$ and there exists at least one compatible node in G

or

Case III: $V \setminus (D_1 \cup D_2 \cdots D_{i-1} \cup D'_i) \neq \phi$ and there exists a node u such that $\forall u' \in N[u]$, $u' \in D_1 \cup D_2 \cdots D_{i-1} \cup D'_i$.

If Case (I) is satisfied then assign $D_i = D'_i$ and we have already constructed the i-th dominating set D_i and next iteration $i+1$ is invoked to generate D_{i+1}.

On the other hand, if $V \setminus (D_1 \cup D_2 \cdots D_{i-1} \cup D'_i) = \phi$ and there is at least a compatible node (Case (II)), then we cannot form the current dominating set and the algorithm reports that $i-1$ is the maximum number of disjoint dominating sets and the execution ceases. This is because there is no node to be included in D'_i and some nodes are still uncovered. Similarly if Case (III) is satisfied then the algorithm terminates and reports $i-1$ as the maximum number of disjoint dominating sets. This is because there is no neighborhood $N[u]$ of u from which we can select a node to cover u.

Let j represent the maximum number of dominating sets returned by our algorithm, that is, the algorithm constructs $D_1, D_2, \cdots, D_j$ disjoint dominating sets.

Algorithm for the Domatic Partition Problem
Input: A UDG $G = (V, E)$
Output: j disjoint dominating sets $D_1, D_2, \cdots, D_j$
1: Compute $t_1(u) =\prec 1/uncov_1(u), dom(u), id \succ$, $\forall u \in V$
2: $D'_1 = \phi$;$i = 1$
3: **while** $i \leq (\delta + 1)$
4: Find the smallest compatible
5: node u in $V \setminus (D'_1 \cup D'_2 \cup \cdots \cup D'_i)$
6: **if** there is such u **then**
7: $D'_i = D'_i \cup \{u\}$
8: **else**
9: **if** Case (II) or Case (III) is satisfied **then**
10: $j = i - 1$
11: break
12: **else**
13: **if** all nodes are incompatible **then**
14: $D_i = D'_i$
15: $i = i + 1$
16: $j = i$
17: $D'_{i+1} = \phi$
18: **endif**
19: **endif**
20: **endif**
21: update $\forall u \in V t_i(u) =\prec 1/uncov_i(u), dom(u), id \succ$
22: **endwhile**

Figure 5.2: An algorithm for the domatic partition problem.

5.4.1 Algorithm Illustration

We illustrate the execution of our algorithm through an example shown in Figure 5.4. Nodes are marked in lower-case letters below the nodes and integers s on the left of nodes denote the dominating sets D_s to which the nodes belong. In the first iteration, we find node a is the smallest compatible node ($t_1(a) =\prec 1/uncov_1(a), dom(a), id(a) \succ=\prec 1/8, 0, id(a) \succ$) in the graph, where $uncov_1(.) = deg(.)$. Select a, add to D'_1 and we update triples $t_1(.)$'s of nodes whose $uncov_1(.)$ and

$dom(.)$ values have been changed. Thereafter, we compute the smallest compatible node in $V \setminus D_1'$ and find b since b's triple $t_1(b) = \prec 1/5, 0, id(b) \succ$ is the smallest. Inserting b into D_1' and repeating the process we compute node c as the smallest compatible node in $V \setminus D_1'$ and insert into D_1'. At this point, we update the triples and find that all nodes have become incompatible which puts an end to iteration 1 and we obtain the first dominating set $D_1 = D_1' = \{a, b, c\}$.

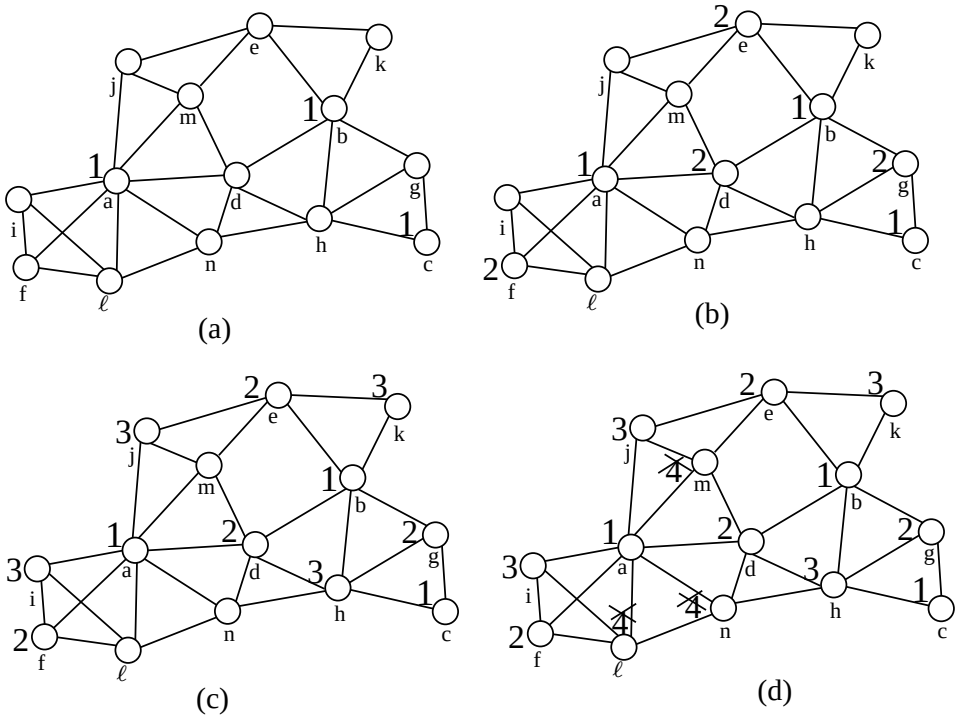


Figure 5.3: Disjoint dominating sets computation. D_1, D_2, and D_3 are shown in (a), (b), and (c) respectively. (d) D_4 cannot be formed. Although the algorithm tries to form D_4, it abandons for not being successful. A cross, 'x' means the formation of D_4 is abandoned.

In the next iteration, we initialize $\forall_{u \in G} uncov_2(u) = deg(u)$ and the $dom(u)$ values are kept from the previous iteration and $D_2' = \phi$. Note that all nodes now

have $dom(.) = 1$. In this iteration, we find d since $t_2(d) =\prec 1/6, 1, id(d) \succ$ is the smallest compatible node in $V \setminus (D_1 \cup D'_2)$ and add to D'_2. After updating the triples, we find the smallest compatible nodes e, f, g in order in $V \setminus (D_1 \cup D'_2)$ and insert them into D'_2. Note that e is chosen before f because although they have the same $uncov_2(e) = uncov_2(f) = 3$ and $dom(e) = dom(f) = 1$ values, we assume $id(e) < id(f)$. This iteration terminates since all nodes are incompatible and it produces the second dominating set D_2 ($D_2 = D'_2 = \{d, e, f, g\}$).

We proceed to the 3rd iteration with $D'_3 = \phi$ and following the algorithm we obtain the smallest compatible nodes h, i, j, k in order in $V \setminus (D_1 \cup D_2 \cup D'_3)$ and put them in D'_3. We stop with the third disjoint dominating set $D_3 = D'_3 = \{h, i, j, k\}$ as the nodes in G become incompatible. As we move to the next iteration ($D'_4 = \phi$), we find ℓ and m as the smallest compatible nodes and update $D'_4 = \{\ell, m, n\}$. Then $V \setminus (D_1 \cup D_2 \cup D'_3) = \phi$ and there exists a number of compatible nodes in G, namely g, k etc. Since the algorithm cannot construct the fourth dominating set D_4, it abandons D_4 and produces as final output the disjoint dominating sets D_1, D_2, and D_3.

5.5 Theoretical Analysis

In this section we present an analysis of our algorithm. First we show that in each iteration m we compute a dominating set D_m, where $m \leq j$ (recall that j is the maximum number of disjoint dominating sets returned by the algorithm) and $D_m \cap D'_m = \phi$, $m, m' \leq j$ and $m \neq m'$.

Lemma 5.5.1 *In G, D is a dominating set if and only if there is no compatible node in G.*

Proof We prove the lemma by contradiction. Assume that D is a dominating set and there exists at least one compatible node v in G. This means either v or some of its neighbors are not covered by any node $w \in D$. Then there can be two situations, either (i) only v is uncovered or (ii) more than one node in $N[v]$ is uncovered. In any case D is not a dominating set in G, a contradiction.

Now we prove the other direction. Again, assume for contradiction that all nodes are incompatible and D is not a dominating set in G. By definition, if all nodes are incompatible then each of them is dominated by some nodes in D, contradicting the fact that D is not a dominating set in G. ∎

In the following, we prove that our algorithm guarantees that it can always generate the first dominating set in UDGs.

Lemma 5.5.2 *For D_1 only Case (I) is satisfied (Case (II) and Case (III) are never reached by the algorithm while computing D_1). Hence $j \geq 1$ (the number of dominating sets returned by the algorithm).*

Proof Assume for contradiction that Case (II) is reached while D'_1 (recall that D'_1 is a temporary set which is assigned to D_1 when D'_1 becomes a dominating set in G) is being computed. Since Case (II) is satisfied, $V \setminus D'_1 = \phi$ and we have at least one compatible node. But this is a contradiction since if $V \setminus D'_1 = \phi$, D'_1 is already a dominating set, and there cannot be any compatible nodes in G.

Similarly assume Case (III) is satisfied while computing D'_1. Then $V \setminus D'_1 \neq \phi$ and there is a node u such that $\forall u' \in N[u]$, $u' \in D'_1$. This is a contradiction because this shows that u as well as all its neighbors belong to D'_1 and hence D'_1 is a dominating set.

It is easy to see that Case (I) must be satisfied if Case (II) or Case (III) is not encountered. Thus the smallest value of j can be at least one and hence for Line 10 $j \geq 1$. ∎

As our objective is to generate disjoint dominating sets, in the following we show that the algorithm can indeed produce dominating sets in G such that any pair of dominating sets are mutually disjoint, i.e., there are no two dominating sets in $D_1, D_2, \cdots, D_j$ which have a common node.

Lemma 5.5.3 *Each D_p is a dominating set and $D_p \cap D_q = \phi$ for $1 \leq p < q \leq j$.*

Proof First we prove that D_p is a dominating set by contradiction. Suppose D_p is not a dominating set at the end of iteration p. Then there is some node v not dominated by any nodes in D_p. That is, $uncov_p(v) \geq 1$ which implies that v is a compatible node. This is a contradiction (from Lemma 5.5.1).

Now we show that $D_p \cap D_q = \phi$ for $1 \leq p, q \leq j$ and $p \neq q$. Without loss of generality, assume $q > p$ and $D_p \cap D_q \neq \phi$. Therefore there exists at least one node in $D_p \cap D_q$. Let $u \in D_p \cap D_q$. Assume $M = V \setminus (D_1 \cup D_2 \cdots \cup D_p)$, where dominating sets $D_1, D_2, \cdots, D_p$ have already been computed. In computing D_q in the q-th iteration, we select nodes from M. Thus if u is picked in D_q then either u does not belong to D_p or D_p is an empty set, yielding a contradiction. ∎

In the following we present the time complexity of our algorithm.

Lemma 5.5.4 *The time complexity of the algorithm is $O(n^3)$.*

Proof The while loop in the algorithm runs for $\delta + 1$ times which could be as large as $O(n)$. For each iteration of the loop, to find the smallest compatible node it takes

O(n) time. Also in each iteration of the loop, for each node that is inserted in the current dominating set we update the triples of all its neighbors and doing so takes O(n^2). Thus the time complexity of the algorithm is O(n^3). ∎

5.6 Simulations

We conducted extensive simulations on random networks to study the performance of the proposed algorithm. In this section, we provide and analyze experimental results regarding domatic partitions produced by our algorithm and compare them with the optimal (maximum) domatic partition. In the experiment, first we obtain UDGs G by generating a set nodes uniformly and randomly in the plane and test the connectivity of G and compute the minimum degree of G. If the underlying graphs are connected then we apply the algorithm to construct disjoint dominating sets and analyze its performance in these random graphs.

5.6.1 Experimental Setup

Experiments are done using our own simulator in Java. In order to evaluate the average performance critically, we choose square fields of different sizes namely, 600m x 600m, 500m x 500m, 400m x 400m, and 300m x 300m. For each of these squares, $n \in \{100, 200, 300, \cdots, 900, 1000\}$ nodes are generated randomly from a uniform distribution. The transmission range of all nodes is set to 80m. For each of the squares and for each value of n we generate 100 different netwrok topologies, and run the algorithm which produces a number of disjoint dominating sets. The choice of different field sizes for the same input size helps generate relatively sparse (for larger squares)

and dense graphs (for smaller squares) to evaluate the algorithm in both situations.

5.6.2 Experimental Results

Setting the transmission range to 80m for all nodes, we apply our algorithm in a 600m x 600m square with $n = 100$ and compute the domatic partition (DP) number (i.e., the number of disjoint dominating sets) and compare with the optimal DP number. Then we generate 100 random graphs successively and for each graph we compute and compare these DP numbers (one by the algorithm and the other is the optimal value). In order to compute the optimal DP value we use an upperbound on the optimal value, which is the minimum degree of G plus 1. Finally, we obtain the average DP number by our algorithm and compare it with the average optimal DP number of all the 100 random topologies. Then the value of n is incremented by 100 (i.e., $n = 200$) and the whole procedure is repeated. We continue incrementing the value of n by 100 each time and repeating the experiment until $n = 1000$.

To obtain more convincing results and to see how the results vary in terms of the DP number, we apply the algorithm and perform the same procedure in squares of sizes 500m x 500m, 400m x 400m, and 300m x 300m, varying n from 100 to 1000 as before and generating 100 different topologies for each value of n. Changing the sizes of squares affects the degrees of nodes, that is, the smaller the area of the field the denser the graphs are and hence the larger the minimum degrees of the graphs. Intuitively the experiment show that for smaller field sizes the DP numbers are greater than that of larger fields for the same values of n.

The results are plotted in Figure 5.4, where the x-axis shows the number of nodes in the generated topology and the y-axis represents the average DP number returned

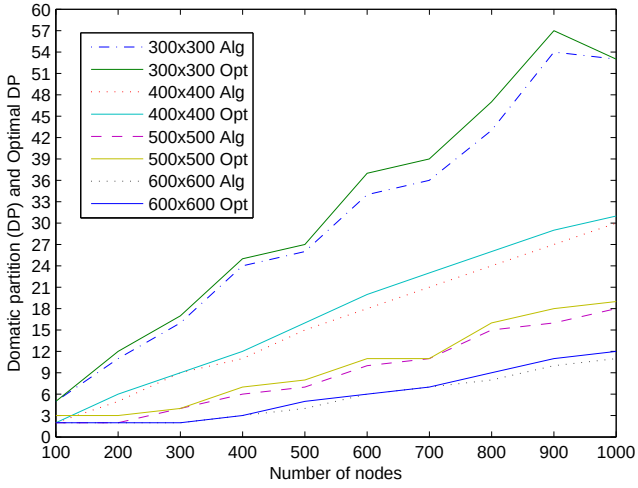


Figure 5.4: Showing the comparison between the average domatic partition (DP) number returned by the algorithm and the average optimal DP number (minimum degree of G plus 1) in different field sizes for each value of $n \in \{100, 200, \cdots, 1000\}$.

by the algorithm (shown as non-solid lines, marked as 'Alg' at the top-left corner of the figure) and the average optimal (maximum) DP number (shown as solid lines, marked as 'Opt' at the top-left corner of the figure). As can be understood from the figure, the bottom line pair (solid and non-solid line) shows the average results of the algorithm and the optimal DP numbers when the input field size is 600m x 600m. For smaller values of n = 100-400, the average DP number almost coincides with the optimal and the values are around 3-5. For higher values of n = 900-1000, the DP number by the algorithm and optimal numbers are in between 9-12. Other pairs of lines are similarly shown with their respective field sizes. In these average cases our experimental results are close to the optimal for all values of n = 100-1000 and in all different sizes of fields. It may be worth mentioning that for smaller values of $n <= 400$, the maximum difference between our average DP value and the optimal

is 2 (additive factor) and for larger values of n the maximum difference is 3 (additive factor).

5.6.3 Cardinalities of Dominating Sets

We also provide some interesting results regarding the average number of nodes in each of the dominating sets. For each run of the algorithm to compute the DP number for a certain value of n, we keep track of the minimum and maximum sizes of dominating sets among the disjoint dominating sets. Since 100 topologies are generated for each value of n, we compute the averages of the minimum and maximum sizes of dominating sets.

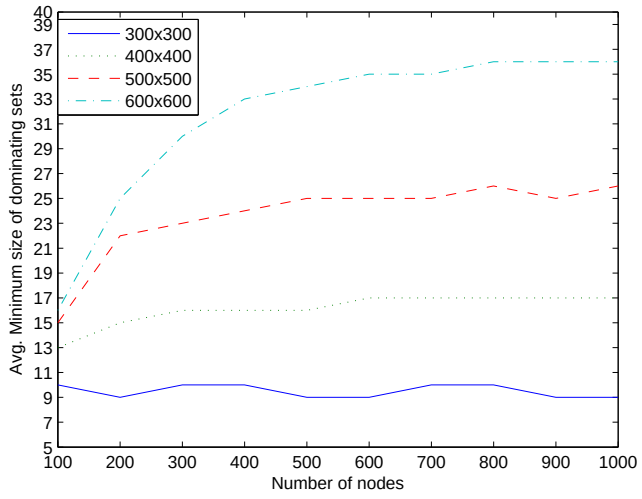


Figure 5.5: Showing the average minimum cardinality of dominating sets found by our algorithm for different values of n.

In Figures 5.5 and 5.6 we plot these values where in the $x-$axis the number of nodes and in the y-axis the average sizes (the average minimum size in Figure 5.5

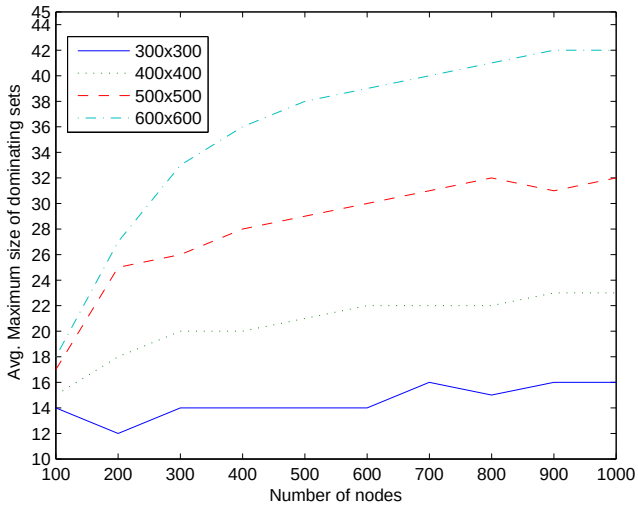


Figure 5.6: Showing the average maximum cardinality of dominating sets found by our algorithm for different values of n.

and the average maximum size in 5.6) of dominating sets are shown. In general, the results show that, for UDGs, the denser the graphs are the smaller the cardinalities of dominating sets. In an intuitive sense, this happens since in a denser graph each node has relatively many neighbors and when it becomes a dominating node it can dominate many nodes. For example, in Figure 5.5, for a 300m x 300m field, the average minimum size of dominating sets varies from $8 - 10$ for all values of n used in our experiments whereas for a 600m x 600m square the average minimum size fluctuates from 16 to 35. Similar results are shown in Figure 5.6 for the average maximum size of dominating sets. A sample output of the algorithm is shown in Figure 5.7.

The mean or average of a value does not reflect the amount of variation among the values. In order to get an idea how the DP numbers spread out over different

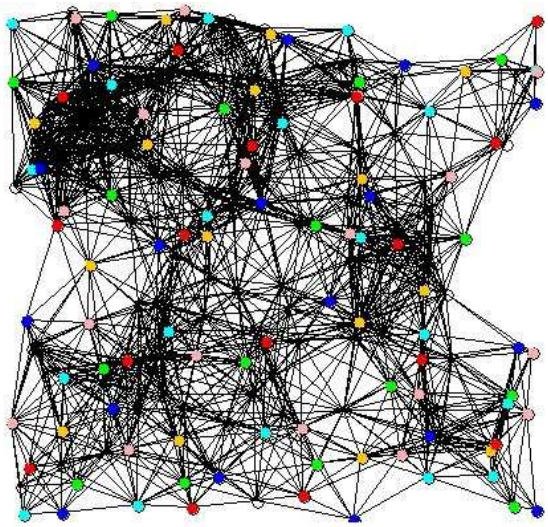


Figure 5.7: A sample output with 200 nodes distributed on 400m x 400m square field where both the optimal DP and the computed DP numbers are 6 (shown in 6 different colors).

values, we compute the standard deviation of the DP numbers (shown in Figure 5.8). The smaller the standard deviation the closer the DP numbers to the average value. As shown in the figure, although the standard deviations for different values of n are smaller (less than 1) and do not vary a lot, they differ in different field sizes. For larger field size (e.g., 600x600) the nodes are generally more sparsely distributed and hence the minimum degree of the underlying graph is relatively small. In such situations, the fluctuations in the values of DP numbers are small for such randomly drawn graphs. Similar reasoning can be applied to smaller field sizes. However, as shown the standard deviations seem to be consistent and reasonably small for all different field sizes.

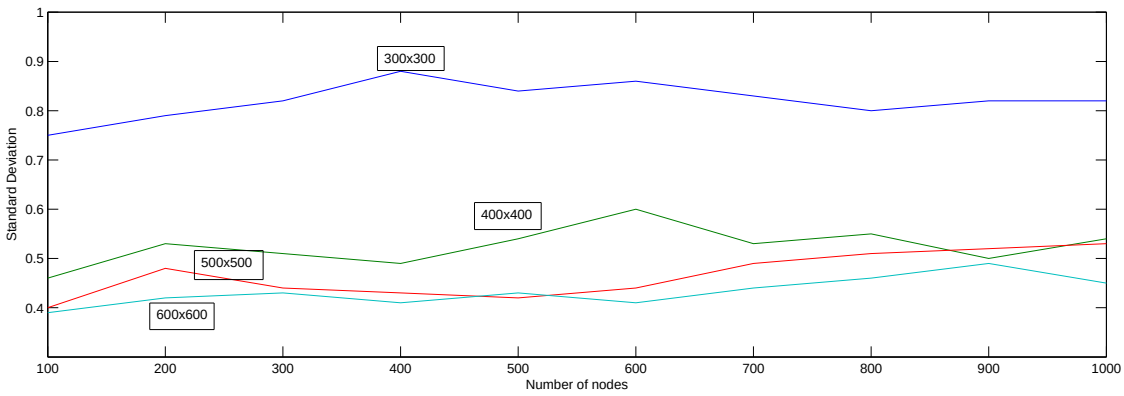


Figure 5.8: Showing standard deviations of DP numbers for different size of fields.

5.6.4 Conjecture about the Maximum Number of Disjoint Dominating Sets

UDGs have a nice property that for any node in the graph, there can be at most five independent nodes in its neighborhood. In other words, for a node u we need at most 5 nodes to cover (dominate) all its neighbors. This result can be further strengthened. In a constant-size neighborhood (e.g., a k-hop neighborhood) UDGs at most a constant number of independent nodes. This is a strong property of UDGs and other *growth-bounded* graphs (which also have the same property). Utilizing this property and depending on the experimental results, we have the following conjecture:

Conjecture 5.6.1 *For UDGs, the domatic partition problem can be solved within a constant factor of the optimal.*

5.7 Conclusion

In this chapter, we have presented an algorithm based on domatic partition for maximizing the lifetime of WSNs. We have provided extensive simulation results to assess the performance of the algorithm where the experiments show 'good' results. However, one intriguing open question is to come up with an algorithm that gives an approximation factor (preferably 'small') for UDGs. We believe that a constant factor algorithm is achievable by utilizing the structural properties of UDGs. For example, a node can have at most five independent nodes in its neighborhood. For UDGs, the only known factor for this domatic partition problem is logarithmic and obtained by a randomized algorithm [58]. Since centralized algorithms do not generally scale up for large networks, distributed algorithms are sought for large wireless sensor networks. Thus it would be challenging to find a distributed solution with a small constant approximation factor for the domatic partition problem in UDGs.

Chapter 6

Target Monitoring

In this chapter, we consider the following target monitoring problem: Given a set of stationary targets $T = \{t_1, \cdots, t_m\}$ and a set $V = \{v_1, \cdots, v_n\}$ of sensors, the target monitoring problem asks for generating a family of subsets of sensors $V_1, \cdots, V_s$ called the *monitoring sets*, such that each V_i monitors all targets. In doing so, the objective of this problem is to maximize $z = s/k$, where k=$\max_{v_j \in V}|\{i : v_j \in V_i\}|$. Maximizing z means generating a large number of monitoring sets and minimizing the number of frequencies of a sensor in these sets.

As energy is one of the main issues of WSNs and sensing consumes energy, maximizing z may have a direct impact in prolonging the lifetime of such networks. Basically, the problem is related to the one we studied in Chapter 4. Unlike finding a family of CDSs, here we generate a family of subsets of nodes (nodes in a subset are not necessarily connected) each of which monitors all the targets (the number of targets is independent of the underlying UDG). However, by maximizing z, we distribute the monitoring responsibilities to all sensors as equally as possible and at the same time, reduce the participations of individual sensors in these monitoring sets.

Towards achieving this goal, a local algorithm is presented. The algorithm is simple and requires each node to know only its 2-hop neighborhood. Furthermore, nodes do not need to know their geographic positions. In short, the main idea is that each node maintains a record (counter, id), where counter is the number of previous monitoring sets in which the node has been selected to monitor the targets. To select sensors for the next monitoring set, for each target, the lexicographically smallest sensor covering it, is selected. We provide special instances where the algorithm obtains an optimum result. As a by-product of the algorithm, it is shown that the size of a monitoring set is at most a constant times the size of a minimum monitoring set when the number of targets is a constant. We present extensive simulation results to evaluate the performance of the algorithm in randomly generated UDGs.

6.1 Introduction

Consider Figure 6.1, where a network of six nodes is shown. Nodes u, w are within their transmission range and can sense a target (shown as a rectangle) as it falls in their sensing range. Node v can also sense the target but none of u or w is v's neighbor. When a node remains active and senses a target in its sensing range, we say the node monitors the target (and the set of all such monitoring nodes forms a monitoring set). After the ad hoc network is formed, nodes begin sensing the environment and depending on the application, such as target tracking, target monitoring, area coverage, and so on, they cooperate by sending and receiving sensed information among their close neighborhood.

However, the more a node engages in sensing targets (in their vicinity) and transmitting, the more energy it consumes. As previously mentioned, in most applications

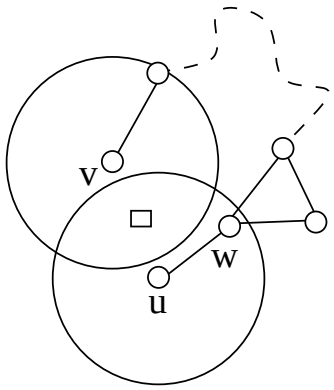


Figure 6.1: A sensor network in which three sensors can sense a target which falls in their sensing range.

a sensor's energy cannot be replenished. However, it is generally expected that sensors will continue to provide information about its sensing targets for a long period of time. This raises the need to judicially utilize the sensors to obtain required information from the sensing field, since otherwise sensors will quickly drain their energy and be unable to function any longer.

An algorithm for WSNs should be energy-efficient in the sense that it ensures proper selection of sensors for sensing and transmission such that no sensors are over or underutilized, and at the same time meets the expected standard of quality in the solutions provided by them. In this chapter, in order to solve the target monitoring problem we propose a 2-local algorithm where the sensors are restricted to know only a very close neighborhood. If a sensor is engaged in sensing (and /or transmitting), it is said to be *active*, in which case it consumes energy, otherwise the sensor is in the *sleep* mode, in which case it does not lose energy.

We distribute the monitoring responsibilities among the sensors in a more balanced way, that is, by having a large number of monitoring sets we increase the chance of

selecting all sensors in the network in these sets, and by minimizing the number of participations of individual nodes in these sets, we reduce the possibility of selecting the same nodes. Therefore, as nodes in the network are put in the active and the energy-efficient sleep mode alternatively, the lifetime of individual nodes is prolonged and hence the lifetime of the network. Another important aspect of the algorithm is the issue of scalability. Since it is a local algorithm where the nodes need to know only their 2-hop neighborhood, it is a suitable candidate for the target monitoring problem in large WSNs.

The rest of the chapter is organized as follows. In Section 6.2 some related work about the problem is described. In Section 6.3 we provide definitions and assumptions that are used throughout the chapter. The algorithm is presented in Section 6.4 and a theoretical analysis of the algorithm follows in Section 6.5. Experimental results are presented in Section 6.6 with conclusion in Section 6.7.

6.2 Related Work

Coverage (also called *monitoring*) has been one of the important topics in WSNs and has received a lot of attention during the past several years [8, 10, 11, 24, 47, 31, 67, 71, 78]. The main goal of most of the work is to devise scheduling algorithms such that individual sensors in the network are assigned time-slots which indicate to them during which time-slots they will be active and during which rounds they will be in the sleep mode. When a number of sensors monitor a certain area or a target, it is generally possible to monitor the area or the target by a *small* subset of them. So, it is redundant to make all the sensors active at the same time for the monitoring. This observation leads researchers to devise efficient algorithms such that at any time only

a small number of sensors are set as active to monitor the area or the targets.

In [8] the authors consider the target coverage problem with the goal to minimize energy consumption. Given a set of targets, they propose an energy-efficient centralized scheme to provide coverage to all the targets by disjoint sets of sensors. They divide the sensor set into disjoint subsets and activate each set to perform the covering task. Basically, their main idea is to generate as many disjoint sets of sensors as possible such that each set covers all the targets (this is similar to the domatic partition problem). However, the algorithm is centralized and does not scale for larger networks.

In [69] the authors address the area coverage problem where their algorithm works in two phases. In the first phase, the algorithm determines how many sensors cover the different parts of the monitored area. Like the algorithm in [8], the second phase allocates sensors into mutually independent sets where some set is active at any time to provide the area coverage. The algorithm is also centralized and the authors do not provide any theoretical results regarding the performance of their algorithm. Simulation results show that the algorithm achieves significant power savings (by generating more disjoint sets of sensors) in performing area coverage.

The work relevant to ours is [9], where the authors consider the target coverage problem by a set of subsets of nodes while the subsets do not need to be disjoint. They call this the maximum set cover (MSC) problem and present two centralized algorithms, one based on linear programming and the other is a greedy algorithm. They do not provide any theoretical analysis of their algorithms and give simulation results to verify their approaches. Here, we consider the target monitoring problem where we focus on generating a number of monitoring sets and at the same time

reducing the number of participations of each node in these generated sets.

A recent result also related to our problem is described in [7], where the authors consider the *monitoring schedule* problem: Given a set of sensors and a set of targets, it is required to find a partition of the sensor set such that each part can monitor all targets. Each part of the partition is used for one unit of time and the goal is to maximize the number of parts in the partition. They present a randomized distributed algorithm which generates at least $(1-\varepsilon)*opt$ $(0 < \varepsilon < 1)$ parts, with high probability, where *opt* is the maximum number of parts in the partition. However, they make the assumption that the nodes must know their geographic positions.

The authors also suggest that by modifying their algorithm they can find a constant approximation factor for the problem and the sensors do not need to know their geographic positions. Our work is related to theirs in the sense that we maximize the number of parts while trying to reduce the use of the same nodes in these parts. Besides, ours is a deterministic algorithm as opposed to their randomized one and the assumption that the sensors know their positions is excluded.

6.3 Preliminaries and Some Definitions

The target set is denoted as $T = \{t_1, \cdots, t_m\}$, where t_i is a target. The set of targets monitored by node u is referred to as $T(u)$. For u and $t \in T(u)$, $T_t(u)$ represents the set of nodes in $N_2[u]$ that monitors the target t (i.e., $T_t(u) = \{t | t \in T(u) \cap T(v), v \in N_2[u]\}$).

Each node u maintains an ordered pair at each iteration i (initially $i = 1$), $p_i(u) = \prec (ct_i(u), id(u)) \succ$, where the first element $ct_i(u)$ (also called the *counter*) denotes the number of monitoring sets in which u has already participated. Initially,

$ct_1(u) = 0$, $\forall u \in V$, and this is incremented by one ($ct_i(u) = ct_i(u) + 1$) each time u participates in some monitoring set. For a subset X, and two nodes $u, v \in X$, we say that node u is lexicographically smaller than node v at iteration i if $p_i(u) < p_i(v)$, i.e., either $ct_i(u) < ct_i(v)$ or $ct_i(u) = ct_i(v)$ and $id(u) < id(v)$. As before, the rank of node u with respect to X, denoted by $r(X(u))$, is the index of u in the lexicographically sorted (ascending order) nodes of X.

A node's transmission and sensing ranges are denoted by TR and SR, respectively. There are a number of reasonable assumptions relating TR and SR and we study two popular ones [7, 25]. In dealing with the target monitoring problem, we consider two situations (i) $TR = SR$ and (ii) $TR = 2 * SR$.

6.3.1 Problem Formulation

We formulate the target monitoring problem in the following way. Given a set of m targets $T = \{t_1, \cdots, t_m\}$ and a set of n nodes $V = \{v_1, \cdots, v_n\}$ such that for each target in T there is at least one node that can monitor it, we would like to find a family $\mathcal{V}$ of subsets $V_1, \cdots, V_s$ such that

i) $\forall i$, V_i monitors all targets in T,

ii) $z = s/k$ is maximized, where k=$\max_{v_j \in V} |\{i : v_j \in V_i\}|$.

6.4 The Algorithm

In this section we present a 2-local algorithm for the target monitoring problem and discuss some problems related to the *myopic* view of localization. We assume that

the transmission range TR is equal to the sensing range SR. As individual nodes execute the same algorithm, we describe what happens to an arbitrary node u in the network.

The algorithm works in iterations starting from iteration 1. At the first iteration ($i = 1$), node u forms its monitored target list $T(u)$ by sensing (or monitoring) the targets in its sensing range. Then u sends two items to all its neighbors which are at most 2-hops away. It sends its list $T(u)$ and an ordered pair $p_i(u) = \prec (ct_i(u), id(u)) \succ$ to $v \in N_2(u)$. Note that initially $ct_1(u) = 0$. Similarly u receives such information ($T(v)$ and $p_i(v)$) from all its neighbors $v \in N_2(u)$. Thus u knows the targets that are monitored by its one and two hop neighbors.

After obtaining $T(v)$ and $p_i(v)$ from all its one and two hop neighbors, u forms the set $T_a(u)$ for each of its monitored target $a \in T(u)$ (recall that $T_a(u) = \{a | a \in T(u) \cap T(v), v \in N_2[u]\}$). In other words, the set $T_a(u)$ refers to the nodes monitoring target a that are at most 2-hops away from u. Then for each $T_a(u)$, u computes its rank $r(T_a(u))$ in this iteration.

If u is the smallest ranked node in any $T_a(u)$, then it becomes active to monitor a, otherwise it goes into the sleep mode. According to the notation, all active nodes in iteration i are represented by V_i which monitor all the targets. If u becomes active in iteration i then its counter is incremented by one ($ct_{i+1}(u) = ct_i(u)+1$), otherwise the counter value remains the same. Node u then sends its $p_{i+1}(u) = \prec (ct_{i+1}(u), id(u)) \succ$ to, and receives $p_{i+1}(v)$ from, $v \in N_2(u)$ and the new iteration $i+1$ begins. As before, node u computes its rank (since $p_{i+1}(.)$ values are updated) for each target $a \in T(u)$. The smallest ranked node for a target a becomes active and starts monitoring a. All smallest ranked nodes in the network form the monitoring set V_{i+1} which monitors

all targets in T and so on. The algorithm is given in Figure 6.4. Preliminary versions of this algorithm have appeared in [35, 36].

For example, if nodes u_1, u_2, u_3 (assume the subscript denotes the respective id) monitor target b (see Figure 6.2), then $|T_b(u_1)| = 3$. At iteration 1, u_1 becomes active by being the smallest ranked node for monitoring b while the other two nodes can go into the sleep mode, then at next iterations 2 and 3, u_2 and u_3 take their turn to be active by becoming the smallest ranked node, respectively. Thus the monitoring sets in iteration 1,2, and 3 are $V_1 = \{u_1\}$, $V_2 = \{u_2\}$ and $V_3 = \{u_3\}$, respectively.

6.4.1 A Problem with Locality

In general, it is assumed that nodes which are in close proximity in the plane sense or observe the same data due to the spatial correlation. For a subset X_t of nodes that monitor the same target t, it is expected that the nodes will be in close proximity. However, the nodes in X_t, although they monitor the same target, can have an arbitrarily long hop distance between each other, while the Euclidean distance may be slightly more than their transmission range. We call this situation the *Locality Effect*, where the nodes monitoring a certain target do not know about each other's monitoring.

In Figure 6.1, the Euclidean distance between u and v is $TR + \varepsilon$ (recall that $0 < \varepsilon < 1$) and they both monitor the same target (shown as a rectangle), whereas their shortest path $d(u, v)$ (in terms of hop distances) can be arbitrarily long.

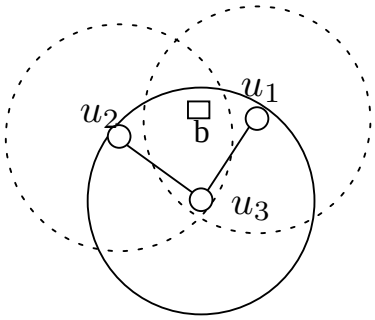


Figure 6.2: Nodes u_1, u_2, and u_3 monitor target b and the monitoring sets are $V_i = \{u_i\}, \forall i$.

6.5 Theoretical Analysis

In this section we give an overview of the theoretical analysis of the algorithm. Consider a target $t \in T$ and let X_t be the set of nodes that monitor it. For any node $u \in X_t$, u knows only whether other nodes, which are within its 2-hop neighborhood (i.e., $N_2[u]$), can monitor t and therefore chooses exactly one among $N_2([u])$ for the monitoring. However, due to the locality effect there can be two nodes $u, v \in X_t$ such that $v \notin N_2(u)$ and both u and v monitor t, and u does not have any clue about v. We would like to determine an upper bound on how many sensors can monitor a target while none of them is aware of the other.

First, we have the following simple observation from the algorithm.

Observation 6.5.1 *The set V_i of all active nodes monitors all targets in T at iteration i.*

As one of the main results, we show that at any round at most five sensors can simultaneously monitor a target, none of which is within the 2-hop neighborhood of the other. In the following, we assume that $TR = SR$.

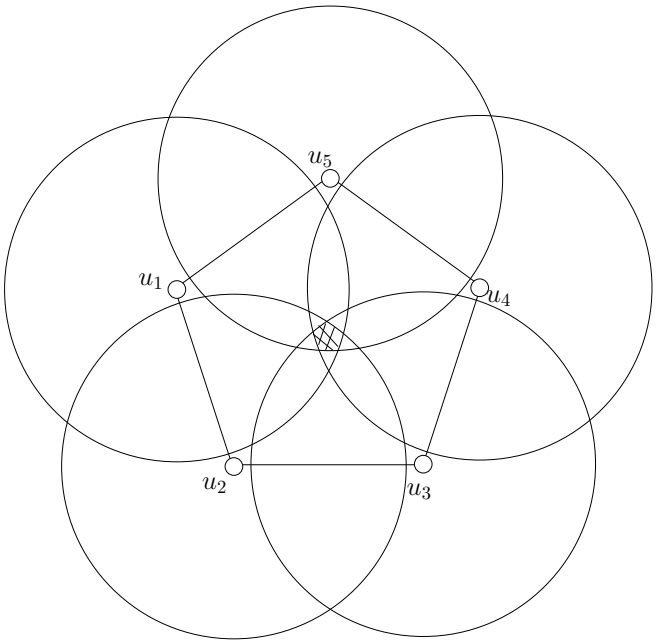


Figure 6.3: For a target in the striped region (intersection of the five disks) we set as active at most five nodes whereas only one node suffices.

Lemma 6.5.2 *For any target $t \in T$, at most five nodes are set as active.*

Proof Assume X_t to be the set of nodes that monitors a target $t \in T$ in some iteration. Suppose for the sake of contradiction that $|X_t| > 5$ and no two nodes $u, v \in X_t$ are within the 2-hop neighborhood of the other. As t is monitored by all nodes in X_t, the Euclidean distance between t and any node $w \in X_t$ must be at most SR. Consider a disk D centered at t with radius equal to SR. All the nodes in X_t must be within the disk D. Since $|X_t| > 5$ and nodes of X_t are in D, there will be at least two nodes $u', v' \in X_t$ whose Euclidean distance must be smaller than TR (we consider TR the radius of D instead of SR, since $SR = TR$). Then u' and v' are direct neighbors to each other, a contradiction. ∎

Input: A UDG $G = (V, E)$ and a set of targets T such that each target is monitored by at least one node.
Output: A family of subsets of nodes $V_1, \cdots, V_s$ such that each V_i monitors all targets in T.
1: $i = 1$
2: Broadcast $p_i(u)$ and $T(u)$ to $v \in N_2(u)$
3: Receive $p_i(v)$ and $T(v)$ from $v \in N_2(u)$
4: Compute $T_a(u) = \{a | a \in T(u) \cap T(v), v \in N_2(u)\}$, $\forall a \in T(u)$
5: /*$T_a(u)$ represents the set of nodes that monitor a*/
6: **For** each round i
7: Compute rank $r(T_a(u))$, $\forall a \in T(u)$
8: **If** $\exists a \in T(u)$ such that
9: $r(T_a(u)) < r(T_a(v))$, $v \in N_2(u)$ **Then**
10: /*Selects the smallest ranked node for each target*/
11: u becomes active
12: **Endif**
13: $i = i + 1$
14: **If** u is active **Then**
15: $ct_i(u) = ct_{i-1}(u) + 1$
16: **Endif**
17: **If** $ct_i(u) \neq ct_{i-1}(u)$ **Then**
18: Send $p_i(u)$ to $v \in N_2(u)$
19: **Endif**
20: Receive $p_i(v)$ from $v \in N_2(u)$
21: **Endfor**

Figure 6.4: A 2-local algorithm for the target monitoring problem.

A tight example of the above lemma is shown in Figure 6.3, where five nodes are put on the vertices of a regular pentagon with side $TR + \varepsilon$. They can all monitor any target t in the striped region S which is the intersection of their respective sensing disks. Note that for any u_i, $u_i \notin N_2[u_j]$ ($i \neq j$ and $i, j \in \{1, 2, 3, 4, 5\}$).

Then we derive the corollary from the above lemma.

Corollary 6.5.3 *If V_i^* denotes the minimum set of nodes monitoring targets at iteration i, then $|V_i| \leq 5m \leq 5|V_i^*|$, where $m = |T|$ is the number of targets given and*

$|V_i^*| \leq m$. ∎

Here we assume that in at each iteration i, we compare the size of the returned monitoring set V_i by our algorithm to the minimum (optimal) sized monitoring set V_i^* of the same iteration.

Now we show how the algorithm performs towards maximizing the value of $z = s/k$. First we need some notation. Let $X_1, X_2, \cdots, X_m$ be subsets of sensors ($X_i \subseteq V$, $1 \leq i \leq m$) that monitor the targets $t_1, t_2, \cdots, t_m$, respectively. Let X_p be the minimum cardinality subset among all X_i's (ties are broken arbitrarily).

We divide each X_i (the nodes in X_i monitor target i) in the following way. Due to the locality effect we can have at most five subsets $X_i(i_1), X_i(i_2), X_i(i_3), X_i(i_4)$, and $X_i(i_5)$ of X_i (i.e., $X_i(i_a) \subseteq X_i$, $1 \leq a \leq 5$, $1 \leq i \leq m$) such that no node in one subset knows whether any nodes in other subsets are monitoring i. In other words, there is no node $u \in X_i(i_a)$ such that $v \in X_i(i_b)$, $a \neq b$ and $v \in N_2[u]$. Let X^*=$\min_{1 \leq i \leq m, 1 \leq a \leq 5} |X_i(i_a)|$. Note that $X^* \geq 1$.

Denote by z_{opt} the optimal value of z and $z_{opt} > 0$, that is, $z_{opt} \geq z = s/k$ for all possible values of s and k. If z_{alg} denotes the value of z obtained by our algorithm then we have the following lemma:

Lemma 6.5.4 *$z_{opt} \leq |X_p|$ and $z_{alg} \geq |X_p|/5$. Hence $z_{alg} \geq |X_p|/5 \geq z_{opt}/5$.*

Proof First we derive the value of z_{opt}. In order to compute the value z_{opt}, notice that there can be at most $|X_p|$ monitoring sets $V_1, V_2, \cdots, V_{|X_p|}$ such that each node in X_p can participate in these monitoring sets exactly once. It is easy to see that for the optimal algorithm to generate the $|X_p| + 1$-st monitoring set, $V_{|X_p|+1}$, there will be at least one node in X_p such that the number of participations of that node in

$V_1, V_2, \cdots, V_{|X_p|}, V_{|X_p|+1}$ is at least two (because of the Pigeonhole Principle). Thus $z_{opt} \leq s/k \leq |X_p|$, where $s \leq |X_p|$ (i.e., at most $|X_p|$ monitoring sets $V_1, V_2, \cdots, V_{|X_p|}$) and $k = 1$.

We now analyze the behavior of the algorithm to find z_{alg}. With each node participating exactly once, the algorithm can generate at least $|X_p|/5$ monitoring sets $V_1, V_2, \cdots, V_{|X_p|/5}$. Hence $z_{alg} \geq |X_p|/5 \geq z_{opt}/5$. ∎

6.5.1 $TR = 2 * SR$

In this section we assume that the transmission range of a sensor is twice its sensing range, $TR = 2*SR$. In the previous subsection it is shown that we can achieve a good approximation factor (a constant factor) for the size of each of the monitoring sets. However, when the number of targets is a constant, the approximation factor is much worse towards minimizing the value of k. This is because of the limited information of the 2-hop neighborhood available to individual sensors and we assumed that the transmission range is equal to the sensing range of a sensor. Here we investigate whether the value of k can be improved by assuming $TR = 2 * SR$. This is a reasonable assumption in the context of coverage (monitoring) problems in WSNs [25]. However, we use this assumption and obtain some useful insights into the target monitoring problem.

Observation 6.5.5 *Let node $u \in V$ monitor $T(u) \subseteq T$ targets. If there is a node $v \in V$, $v \neq u$, such that $T(u) \cap T(v) \neq \phi$, then $v \in N(u)$.*

Proof Suppose for the sake of contradiction that $v \notin N(u)$. Since $T(u) \cap T(v) \neq \phi$, that means v can monitor a target which u can monitor. In this case the Euclidean

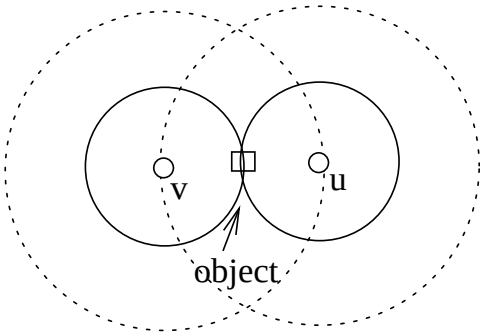


Figure 6.5: If $T(u) \cap T(v) \neq \phi$ then $v \in N(u)$.

distance between u and v can be at most twice their sensing range (see Figure 6.5, where dotted and solid circles represent the transmission and the sensing ranges, respectively) contradicting the fact that v is not a direct neighbor of u. ∎

The above observation implies that if u monitors t then u knows exactly which other nodes are also monitoring t and moreover, all such nodes are u's direct neighbors.

Lemma 6.5.6 *Under the assumption that $TR = 2 * SR$, for any target $t \in T$, exactly one node is set as active.*

Proof The proof is similar to the proof of Lemma 6.5.2. Consider a target $t \in T$ and let X_t be the subset of sensors that monitors it. There is no locality effect as evident from Observation 6.5.5. So each node in X_t knows whether all other nodes in X_t are monitoring t. Hence the node with the smallest rank in X_t will be active to monitor t. ∎

In the following, we show that the algorithm obtains optimal results in special instances. Let $X_1, X_2, \cdots, X_m$ be the subsets of sensors that monitor the targets $t_1, t_2, \cdots, t_m$, respectively, where $X_i \subseteq V$ in some iteration. Let X_p be the minimum

cardinality subset among all X_i's (ties are broken arbitrarily). If $X_i \cap X_j = \phi$, $i \neq j$ and $1 \leq i, j \leq m$, that is, a sensor monitors exactly one target, then we have the following lemma.

Lemma 6.5.7 *With the above assumption and $TR = 2 * SR$, we have $z_{alg} = z_{opt}$.*

Proof Let $X_p = \{u_1, u_2, \cdots, u_{|X_p|}\}$ denote the subset of sensors that monitor p (recall that X_p has minimum cardinality among $|X_i|$'s). The optimal algorithm can produce at most $|X_p|$ monitoring sets $V_1, V_2, \cdots V_{|X_p|}$ with each node in X_p participating exactly once. Since X_p has the smallest cardinality, for the optimal algorithm to produce the next monitoring set $V_{|X_p|+1}$, at least one of the nodes in X_p must be selected in $V_{|X_p|+1}$. So, $z_{opt} = |X_p|/1 \geq (|X_p| + 1)/2$.

We see how the algorithm behaves towards finding z_{alg}. Since there is no locality effect (all nodes $u_1, u_2, \cdots, u_{|X_p|}$ are aware of each other about monitoring p), the algorithm sets as active exactly one node from X_p (say node u_j is selected in some iteration). Since $X_p \cap X_i = \phi$, for $i \neq p$ and $1 \leq i \leq m$, no other nodes from X_p will be selected (active) in that iteration. Therefore, at any iteration exactly one node is set as active from X_p and hence we have $V_1, V_2, \cdots V_{|X_p|}$ monitoring sets.

Now we show that any node $u_j \in X_p$ is exactly used once in $V_1, V_2, \cdots V_{|X_p|}$ monitoring sets.

Suppose for contradiction that a node u_j is used in monitoring sets V_r and V_w, where $1 \leq r, w \leq |X_p|$ and $r < w$. After being selected first in V_r, u_j's $ct(u_j)$ value is increased by one. If u_j is selected (from X_p) again in V_w, that means its rank is smaller than other nodes in X_p. Since $w \leq |X_p|$ and exactly one node is selected from X_p, there is at least some node in X_p which has not been selected in previous

monitoring sets $V_1, V_2, \cdots, V_{w'<w}$. If this has not been selected in previous monitoring sets its rank will be smaller than u_j's, a contradiction. The claim follows. ∎

6.6 Simulations

We conduct extensive simulations on randomly generated UDGs to study the performance of the proposed 2-local algorithm. In this section, experimental results are presented regarding (i) maximizing $z = s/k$ and (ii) the sizes of monitoring sets $V_1, V_2, \cdots, V_s$. In the experiments, first we distribute a set of $m \in \{10, 20, 30, 40, 50\}$ targets randomly and uniformly in a square field of size 400m x 400m. Thereafter, a set of $n \in \{100, 200, 300, 400, 500\}$ nodes is generated (uniformly and randomly) in the plane from which a UDG is formed. The algorithm then constructs monitoring sets for the targets.

6.6.1 Experimental Results

Experiments are done using our own simulator in Java. Setting both the transmission and the sensing range to 60m for all nodes, we apply the algorithm in a 400m x 400m square with $n = 100$, $m = 10$ and first compute the minimum cardinality set X_p. Recall that given $t_1, t_2, \cdots, t_m$ targets, X_p is the minimum cardinality set among $X_1, X_2, \cdots, X_m$, where $X_i \subseteq V$ denotes the set of nodes that monitors the target t_i, $(1 \leq i \leq m)$. Then the monitoring sets $V_1, V_2, \cdots, V_{|X_p|}$ are generated.

In the experiments, we set the optimal value of z, $z_{opt} = s/k = |X_p|/1$, since the optimal algorithm can generate at most $|X_p|$ monitoring sets with $k = 1$. In order to find the value of z_{alg}, we figure out the maximum frequency k of a node in X_p in

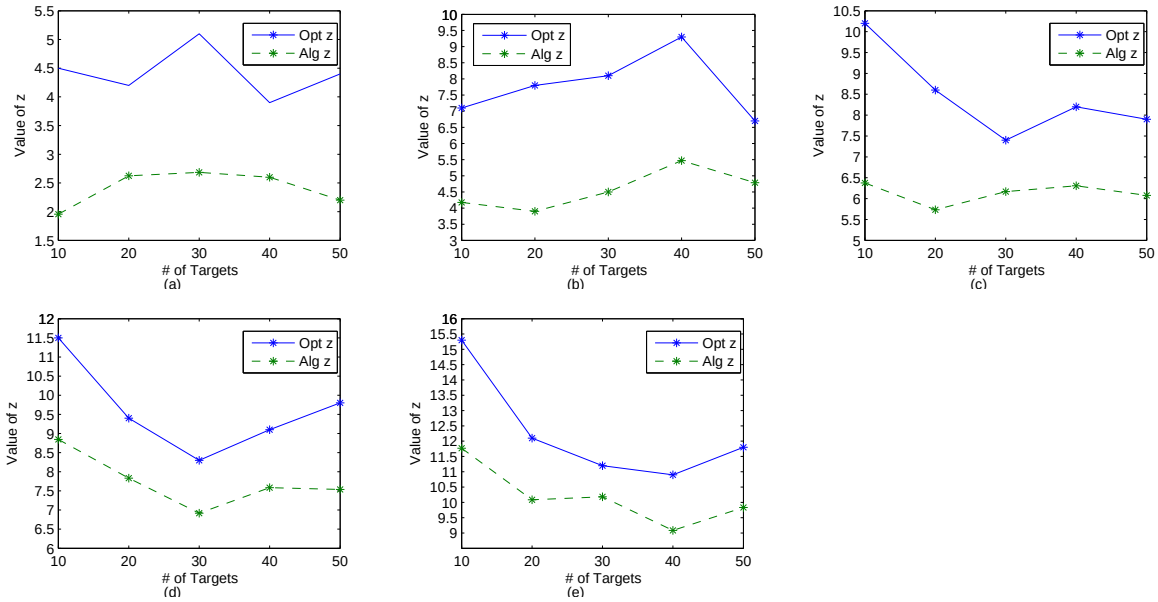


Figure 6.6: Average z values (optimal z_{opt} values by solid lines and z_{alg} by dashed lines) for (a) 100 (b) 200 (c) 300 (d) and (e) 500 sensors.

these monitoring sets and obtain $z_{alg} = |X_p|/k$. After generating 100 random graphs successively with the same number ($m = 10$) of targets, the size of the minimum cardinality set $|X_p|$ and the maximum frequency k of a node in the monitoring sets are obtained. Hence z_{opt} and z_{alg} for each graph are computed. Finally the average values of z_{opt} and z_{alg} are determined.

Keeping $m = 10$, we increment the value of n by 100 each time and repeat the whole procedure until $n = 500$. In this way, we obtain the average values of the parameters (z_{opt} and z_{alg}) for each value of n. Thereafter, for each value of $m \in \{20, 30, 40, 50\}$ and for each value of $n = \{200, 300, 400, 500\}$, the experiment is run 100 times and the averages of z_{opt} and z_{alg} are determined.

The results are plotted in Figure 6.6, where the x-axis shows the number of targets in the field and the y-axis represents the average values of z_{opt} and z_{alg}. Optimal values

z_{opt} are shown in solid lines and the values z_{alg} by the algorithm are shown in dashed lines. In these average graphs the experimental results found are much better than the theoretical bounds established in Lemma 6.5.7. Note that the average z_{alg} values are within a small constant (at most 3) of the optimal average z_{opt} values. As is obvious from the figures the results are even better and consistent in dense graphs (we assume, having 500 sensors in a square of 400m x 400m is denser than having 100 or 200 sensors in the same field). This is intuitive because having more sensors in small a area reduces the locality effect and sensors are likely to be aware of other sensors monitoring the same object.

Finally, as a by product of the algorithm we also consider the sizes of individual monitoring sets. We compute the cardinality of each monitoring set V_i and compare it with the optimal size of the monitoring set V_i^*. To find the optimal value $|V_i^*|$, we have used the number of targets as the value of $|V_i^*|$, that is, $|V_i^*| = m$. We compute the size of each monitoring set V_i generated by the algorithm and find that on average it is 3.16 times the size of V_i^*.

In previous figures, we have shown the average z_{alg} values for different number of sensors and targets. In Figure 6.7, we plot standard deviations of z_{alg} to see how the values deviate from the average z_{alg}. The smaller standard deviations (less than 0.7) for all values of n are an indication that most z_{alg}'s tend towards the mean. For large n, the standard deviation seems to be lower than that of smaller n. This is because the locality effect, in general, is much less for a large number of sensors distributed in a relatively smaller area.

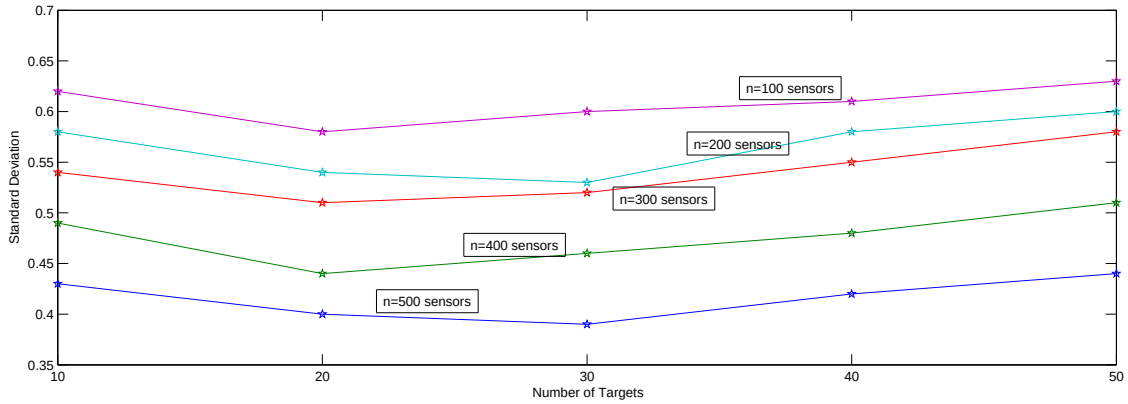


Figure 6.7: Showing standard deviations of z_{alg} values for different number of targets and sensors.

6.7 Conclusion

In this chapter, we have presented a simple 2-local algorithm to compute a family of monitoring sets such that each set monitors all the targets given in some field. Generating a number of different monitoring sets with the purpose of minimizing the frequency of participations of a node in these sets increases the probability of prolonging the lifetime of WSNs for such applications. Our algorithm is simple and strictly local, since each node only requires the knowledge of its two hop neighborhood and thus can be easily applicable to WSNs.

We provide theoretical results about the sizes of monitoring sets, determine bounds on the value of $z(z = s/k)$ when the transmission range is equal to and twice the sensing range. We provide extensive simulation results to assess the performance of the algorithm. The experiments in random graphs show much better results than the theoretical bounds established in the chapter.

Chapter 7

Self Protection

Since sensors provide monitoring to targets, it is often necessary to give them a level of protection such that sensors can take certain actions when they are themselves the subject of attacks. One natural idea is to monitor sensors by their neighbors such that these neighbors can inform the base station when monitoring sensors malfunction or are in danger.

Keeping this in mind, the p-self-protection subset problem has been formally introduced in [79]. The p-self-protection subset problem is defined to be a subset of sensors such that for any sensor in the network there are at least p active sensors from the subset that monitor it. In this chapter, we study the p-self-protection subset problem.

The authors in [79] first show that finding a minimum 1-self-protection subset is NP-hard and then give an $O(\log n)$-approximation algorithm. Later, with a view to obtaining a constant factor approximation ratio, centralized and distributed algorithms to solve the general p-self-protection subset problem are presented in [77]. Here, first we present a counterexample showing that the algorithm in [77] fails to find a p-self-protection subset in the network. Thereafter a distributed algorithm

for the p-self-protection subset problem is given which achieves a constant factor approximation ratio. To the best of our knowledge, this is the first constant factor approximation ratio to the p-self-protection subset problem. The distributed algorithm runs in $O(\log^* n)$ time, where n is the total number of sensors in the network.

7.1 Introduction

As pointed out earlier, once deployed with a limited amount of energy, nodes in sensor networks continue to spend energy by sensing, gathering, and distributing data from the environment. Among myriad of applications, sensor networks are used for protection and surveillance of many different environments such as museum monitoring, military surveillance, target tracking, and so on. More specifically, nodes carry out protection and monitoring activities by collecting and sending timely signals to a base station to take appropriate measures in case some unusual events occur to the targets they monitor.

Although sensors are often used for protection of targets as mentioned, there are situations when it is vital to monitor the sensors as well. The issue of monitoring sensors arises because battery-powered sensors are vulnerable to damage, failure and attacks. If sensors themselves are incapable of performing monitoring, then how could we protect targets (or objects, they are used interchangeably)? It is therefore important to ensure that sensors are active and functioning as desired. One of the ways this can be done is to require neighboring sensors to monitor the conditions of each other.

Imagine a scenario where among the set of sensors there is a single sensor which is monitoring a certain object. If, for some reason, this sensor malfunctions or runs out

of energy then the object will be left unprotected. However, if there were some other neighboring sensors which could monitor this particular sensor then any status of that sensor (whether running out of energy or in danger) could be relayed by them to the base station so that proper action could be taken. For example, the sensor may be replaced by additional sensors, if it does not function properly. This scenario raises the issue that sensors themselves require a certain level of protection. The problem of protecting the sensors by other sensors is called *self-protection*. In its simplest form, a sensor network is self-protected if all sensors are protected by at least one *active* sensor (other than itself), where active sensors are sensors which monitor their neighbors by being active. In general, a WSN is p-self-protected if at any moment for any sensor there are at least p active sensors (other than itself) that can monitor it. This problem has been formally introduced and investigated by Liu et. al [79].

The rest of the chapter is organized as follows. Section 7.2 describes related work. In Section 7.3 we define the p-self-protection subset problem, show a counterexample to the work in [77], and present a distributed algorithm to solve the problem. We provide a theoretical analysis of our algorithm in Section 7.4 followed by simulation results in Section 7.5. Conclusion is provided in Section 7.6.

7.2 Related Work

Since sensors are power-limited and monitoring requires sensors to be active all the time thereby consuming valuable energy, finding a 'small' number of sensors to do the monitoring has been one of the main objectives of research in this regard. This small subset of active sensors are used to provide protection for every location of the area, in which they are deployed, and at the same time other sensors can be put in

the energy-efficient sleep mode to conserve energy. A number of coverage problems, including protection of certain static objects with a required degree of fault-tolerance, and methods to solve them have been surveyed by Wu et. al [10], where the main focus is on the area coverage problem.

A generalization of the area coverage problem called the k-coverage, i.e., whether every point is covered by at least k sensors, is studied by Kumar et el [43]. The k-coverage problem is directly related to the fault-tolerance issue in WSNs. This is because k-coverage basically provides fault-tolerance in the network since failure of at most $k' < k$ monitoring nodes to monitor a particular area will not leave that area uncovered. Gui et al [29] propose algorithms for area coverage problems considering fault-tolerance, energy-consumption and quality of coverage.

A shift from the coverage problem to the problem of protecting the sensors themselves was first suggested by Liu et al [79]. In [79] the authors prove that finding minimum 1-self-protection subset (that is, finding a subset of nodes of minimum cardinality such that each node in the network is protected by at least one other node) is NP-complete through a reduction from the set-cover problem [26]. As a consequence they give a centralized algorithm for this problem and achieve a $2(1 + \log n)$ approximation ratio, where n is the total number of sensors. That is, their algorithm finds a 1-self-protection subset which is at most $O(\log n)$ times the cardinality of the minimum 1-self-protection subset. However, they do not give any algorithm for the general p-self-protection subset problem. The authors also show that focusing only on the quality of area or object covering does not necessarily make the sensors to be self-protected. Subsequently, a probabilistic definition of the self-protection subset (a

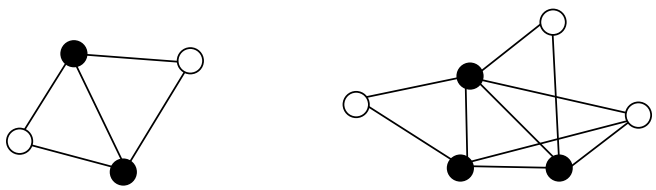
protection is called the δ-p-self-protection if the probability that a node is not p-self-protected is less than δ), as well as two randomized algorithms to solve the problem in a distributed fashion, are given in [79].

Later, the problem of finding a minimum p-self-protection subset was thoroughly studied along with some of its variants such as requiring that the protecting sensors be also connected, allowing sensors with varying sensing ranges, and so on, by Wang et. al [77]. In their paper, the authors claim that they have two constant factor approximation algorithms (in both centralized and distributed settings) for this problem. Unfortunately, their claim is in error. We provide a counterexample to show an instance of a graph where the p-self-protection algorithm does not form a subset that p-protects the nodes in G. They discuss other variants of the problem, namely, how p-self-protection can be achieved when the sensing radii of the sensors are not the same, or generating a certain number of p-self-protection subsets and activating them one after the other.

Here we investigate the p-self-protection problem. By presenting a counterexample of the work of Wang et. al [77], we provide a distributed algorithm to solve the p-self-protection problem. To the best of our knowledge this is the first distributed constant factor approximation algorithm for the p-self-protection problem. This result is significant in two ways: first, it achieves a constant factor which is better than the previous (logarithmic factor) results and second, it is distributed. The earlier version of the algorithm appeared in [34].

7.3 Minimum p-self-protection Subset Problem

As mentioned before, a WSN is p-self-protected if, for any sensor, there are at least p neighboring active sensors (a sensor is not a neighbor to itself) that monitor it. When the network is modeled as a UDG, $G = (V, E)$, the minimum p-self-protection subset [77] is defined as a subset $P \subseteq V$ such that the network is p-self-protected and the cardinality of P is minimized. See Figure 7.1, where the set of black nodes form a 1-self-protection (Figure 7.1 (a)) and a 2-self-protection (Figure 7.1 (b)) subsets. As mentioned before, it is proved in [79] that computing a minimum 1-self-protection subset is NP-hard, which implies that the general minimum p-self-protection subset is also NP-hard. From now on instead of using 'p-self-protection' we use 'p-protection'.



(a) 1−self−protected (b) 2−self−protected

Figure 7.1: Minimum 1-self-protection and 2-self-protection subsets.

In the following, after describing the algorithm of Wang et. el, we present a counterexample which makes the p-protection algorithm fail to find a p-protection subset.

7.3.1 Wang et. al [77] Algorithm: p-protection Subset Problem

The algorithm in [77] for the general minimum p-protection subset problem is now described. In the first iteration ($k = 1$), the algorithm [77] first finds an MIS M_1 based on the ranks (each node u is assigned a unique integer from $[1, n]$ called its rank, $r(u)$) of the nodes, where a node in selected to be in M_1 if it has the largest rank among all its neighbors. After M_1 is formed, each node not selected in M_1 is then assigned a unique rank $r(v) = r(v) + k * n$, where initially k is assigned 1 and incremented by one at each iteration, and the ranks of the nodes belonging to that MIS are not changed. Then in the next iteration($k = 2$), the algorithm generates a new MIS M_2 from the nodes not selected in M_1 based on their ranks.

Thereafter, in the 3rd iteration, ranks are updated by the equation $r(v) = r(v) + 3*n$ for those nodes which did not belong to M_1 and M_2 and a new MIS M_3 is formed. This procedure of generating new MISs is repeated p times, and it is claimed that this algorithm finds p MISs. Finally, as this algorithm continues, if a node $u \in M_i$, $1 \leq i \leq p$ has fewer than p neighboring nodes in $\bigcup_{i=1}^{p} M_i$, then u randomly selects a neighbor $v \notin \bigcup_{i=1}^{p} M_i$ to protect u to such that u becomes p-protected. According to [77], the union of all M_i's and the selected neighbors form the desired p-protection subset.

In order to prove this claim (Theorem 2 in [77]), it is stated that for any node $u \in \bigcup_{i=1}^{p} M_i$, u has at least $p-1$ protectors from $\bigcup_{i=1}^{p} M_i$ since it has been protected by MIS nodes in every iteration except the iteration in which it is selected. Then if u has only $p-1$ neighboring nodes in $\bigcup_{i=1}^{p} M_i$, it just selects an arbitrary neighbor v, which did not belong to any of the M_i's, to protect itself.

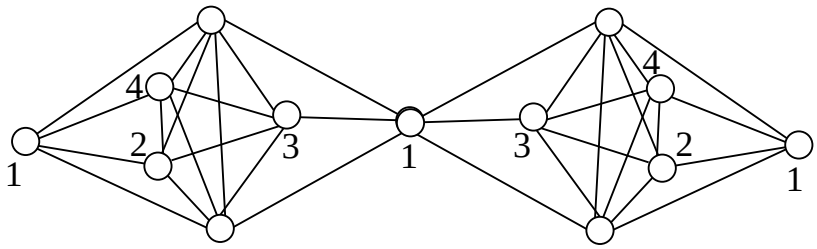


Figure 7.2: Counterexample to the proof of Theorem 2 [77].

However, this statement is not correct. Figure 7.2 shows a counterexample for the p-protection subset problem, where $p = 4$. Assume that the algorithm finds the first MIS M_1 consisting of nodes shown with label 1. According to the algorithm, another MIS M_2 can be found consisting of the nodes that do not belong to M_1. However, in the example, there is no way to form a second MIS M_2 from the remaining nodes $V \setminus M_1$ because any attempt to construct the second MIS without using nodes from the previous MIS will fail.

More specifically, if we want to form the second, the third or the fourth MISs M_2, M_3 or M_4, each will consist of exactly two nodes (shown with label 2, label 3, and label 4, respectively) and none of which is an MIS of the whole graph. This means that in any case, either the middle label 1 node (when we form M_2 and M_4) or the label 1 nodes on the left and on the right (for M_3) are not protected by any nodes. Furthermore, according to [77], for all nodes $u \in \bigcup_{i=1}^{p} M_i$, u must have at least $p-1$ protectors from $\bigcup_{i=1}^{p} M_i$, given that u has already been protected by MIS nodes in every iteration except the iteration in which it is selected as a node in MIS. However, in the example of Figure 7.2, the middle label 1 node is not protected by $p - 1 = 3$ nodes. The authors of the paper [77] agreed that the counterexample we provide is correct [73].

7.3.2 Solution: 1-protection Subset Problem

For the minimum 1-protection subset problem we present a distributed algorithm that gives a constant approximation ratio. Basically our algorithm is almost the same of [77]. However, we give a different proof for the approximation factor. First, nodes distributedly construct an MIS, S (following the algorithm in [66]). Thereafter, each node in S arbitrarily selects one of its neighbors to protect it. Let the set of selected nodes be denoted as S'. All nodes in $S \cup S'$ are set as active. Let the set of active nodes be denoted by A_1. We claim that A_1 forms a valid 1-protection subset. Let A_1^* denote the minimum 1-protection subset. The constant approximation ratio is obtained as follows.

Theorem 7.3.1 *For the minimum 1-protection subset problem, the distributed algorithm finds a valid 1-protection subset A_1 and has size at most 10 times the minimum 1-protection subset A_1^*, that is, $|A_1| \leq 10|A_1^*|$.*

Proof First, we show that A_1 provides 1-protection to the nodes in G. Notice that $A_1 = S \cup S'$. Consider any node u. If $u \notin S$ then it must have a neighbor from S. This is because S is an MIS. So u is 1-protected by some nodes in S. Secondly, if $u \in S$ then it does not have any neighbor that is in S. However, it chooses an arbitrary neighbor from $u' \in N(u)$ to protect itself. Therefore, u is protected by the node it selects.

Each node in S selects an arbitrary node, so we have $|A_1| \leq 2|S|$. In order to form the minimum 1-protection set A_1^*, the optimal algorithm must provide 1-protection to the nodes in S. Since any node $\notin S$ can have at most 5 S nodes in its neighborhood, the optimal algorithm must choose at least one node to provide 1-protection to at most 5 S nodes. So, $|A_1^*| \geq \frac{|S|}{5} \geq \frac{|A_1|}{2*5} = \frac{|A_1|}{10}$ and the claim follows. ∎

It is intersting to note that a CDS is also a 1-self protection subset unless the CDS is a singleton.

7.3.3 Distributed Algorithm $\mathcal{D}$: $p > 1$-protection Subset Problems

In this section, we present a distributed algorithm for the general p-protection subset problem, where $p > 1$. Let A_p and A_p^* denote the p-protection subset produced by the algorithm and the minimum p-protection subset, respectively.

p-protection

First, run the distributed MIS [66] algorithm on G and let S be the set of nodes returned by the algorithm, that is, S is an MIS in G. Nodes in S are set as active. Let $N^q(u)$ denote a set of at most q nodes selected by a node u from its neighborhood, $N^q(u) \subseteq N(u)$. We take advantage of a well-known property of UDGs towards finding p-protection subset. In a UDG, a node u requires a set $N^5(u)$ to protect all nodes in $v \in N(u) \setminus N^5(u)$. Specifically, for any node $v \in N(u) \setminus N^5(u) \neq \phi$, there exists a node $w \in N^5(u)$ such that $vw \in E$. We informally describe the algorithm in the following:

Consider a node $u \in S$. It provides 1-protection to its neighbors $N(u)$. Then it finds a subset $N^{5p}(u)$ to provide the required p-protection to nodes $N[u]$. We need a little explanation of why we choose $N^{5p}(u)$ nodes to provide p-protection to nodes $N[u]$. First note that by selecting $N^{5p}(u)$, u becomes at least p-protected, since $|N(u)| \geq |N^{5p}(u)| \geq p$. Second, the nodes $N(u) \setminus N^{5p}(u)$ are guaranteed to have at least p-protection. Also each node $u' \in N^{5p}(u)$ in the selected node set $N^{5p}(u)$ is

Algorithm $\mathcal{D}$ for p-protection subset
Input: An undirected graph G and an integer parameter p
Output: Protection set A_p
1: Run Distributed MIS Algorithm [66]
2: Let S be the MIS found
3: **If** $u \in S$
4: **If** $p = 1$
5: Select $v \in N(u)$ to protect u
6: Set u and v active
7: **Endif**
8: **If** $p \geq 2$
9: Select a set N^{5p} of at most $5p$ nodes
//N^{5p} provides protection to nodes in $N[u]$
10: Set $N^{5p} \cup \{u\}$ as active
11: **EndIf**
12: **EndIf** //All active nodes form the p-protection subset A_p.

Figure 7.3: Algorithm $\mathcal{D}$ for the p-protection subset.

provided at least $p-1$-protection by the nodes $u'' \in N^{5p}(u)$, $u' \neq u''$. Since u provides protection to u', u is at least p-protected. Therefore, in order to guarantee full p-protection to all nodes $N[u]$, u selects $N^{5p}(u)$. Let S' represent the set of selected nodes.

All nodes $A_p = S \cup S'$ in the graph are set as active and they constitute the solution of the $p-$ protection subset problem. In Figure 7.3, we present the distributed algorithm $\mathcal{D}$ for the p-protection algorithm that is executed at each node u.

7.4 Theoretical Properties of Algorithm $\mathcal{D}$

In this section we provide the correctness of the algorithm, that is, we prove that the algorithm produces a p-protection subset for all the nodes in the underlying graph.

Lemma 7.4.1 *Active nodes in G constitute a p-protection subset to the nodes in G.*

Proof Consider a node $u \in V$. Then we have the following three cases:

Case 1: Node u is not active. Then it must have a neighbor $u' \in S$. Since u' selects $N^{5p}(u')$ to provide protection to its neighbors, $u \in N(u')$ must have at least p active nodes protecting it. In fact, u is at least $p+1$ protected since it is also protected by u'.

Case 2: Node u is active and $u \in S'$. Then it must have a neighbor node $u' \in S$ and $u \in N^{5p}(u')$. Therefore, u gets protected by at least $p-1$ nodes from $N^{5p}(u')$. However, u is at least p-protected since its neighbor u' also protects u.

Case 3: Node is active and $u \in S$. Since u selects a subset $N^{5p}(u')$ of at most $5p$ neighbors to provide p-protection to its neighbors, u is at least p-protected (recall that $|N(u)| \geq p$). ∎

In the following we prove the main result of this chapter.

Theorem 7.4.2 *The size of the set A_p ($p > 1$) returned by the algorithm is at most $25 + \frac{5}{p}$ times the optimum solution A_p^*.*

Proof According to the algorithm, each node $u \in S$ selects a subset $N^{5p}(u)$ of at most $5p$ nodes in its neighborhood to provide the required p-protection to its neighbors. Recall that a node $u' \in V \setminus S$ can have at most 5 S nodes in its neighborhood, that is, $|N(u') \cap S| \leq 5$. Since each of these 5 nodes can select at most $5p$ nodes, we have $|A_p| \leq |S| + 5p|S| = (5p+1)|S|$.

In order to form the minimum p-protection set A_p^*, the optimal algorithm must provide p-protection to the nodes in S. Since a node $\notin S$ can have at most 5 S nodes

in its neighborhood, the optimal algorithm must select at least p nodes to provide p-protection to at most 5 S nodes. Therefore,

$$\begin{aligned} |A_p^*| &\geq p\frac{|S|}{5} \\ |A_p^*| &\geq p\frac{|A_p|}{5(5p+1)} \\ &\geq p\frac{|A_p|}{25p+5} \\ |A_p| &\leq (25+\frac{5}{p})|A_p^*| \end{aligned}$$

∎

7.4.1 Time and Message Complexities

According to our model $\mathcal{LOCAL}_S$, exchanging indices and degrees with direct neighbors can be done in a constant time [61]. Line 1 in the algorithm $\mathcal{D}$ constructs a distributed MIS [66] in a UDG which requires $O(\log^* n)$ time [66], where n is the number of nodes in G. The algorithm in [66] can be applied to any growth bounded graph, where growth boundedness means that there is a constant number of independent nodes in some fixed hop neighborhood. Lines 2-12 of algorithm $\mathcal{D}$ can be executed in constant time since nodes only perform local computation [61] in their constant hop neighborhood. Thus algorithm $\mathcal{D}$ needs $O(\log^* n)$ time.

Each node broadcasts a single message containing its index and degree to its direct neighbors. As mentioned in [66], the algorithm for generating an MIS requires at most $O(|E| \log^* n)$ messages. Also nodes in S inform their selected neighbors to become active by sending them a message which totals at most $O(n)$ messages. Thus the message complexity of algorithm $\mathcal{D}$ is $O(|E| \log^* n)$.

This suggests the following theorem:

Theorem 7.4.3 *For UDGs, the p-protection subset problem can be solved within a constant factor of the optimum, in $O(\log^* n)$ time and $O(|E| * \log^* n)$ messages.* ∎

7.5 Simulations

We conducted extensive simulations on random UDGs to study the performance of our p-protection algorithm. In this section, we provide and analyze experimental results and show how the results differ from optimality. Specifically, we carry out the experiments for $p = 1, 2$ and 3. By placing nodes uniformly and randomly in the plane, we constructed the UDGs. If the underlying graphs are connected we apply the algorithm to construct p-protection subsets.

7.5.1 Experimental Setup

Experiments are done using our own simulator in Java. In the simulation, we have chosen a square field of size 500m x 500m, where $n \in \{100, 200, 300, 400, 500\}$ nodes are distributed. The transmission range of all nodes is set to 60m. For each value of n, we generate 100 different randomly connected topologies. For each value of n and each topology we run the algorithm which generates $p = 1, 2$ and 3-protection subsets for the topology.

7.5.2 Experimental Results

After generating a connected random topology with $n = 100$, we apply the algorithm and compute the sizes of p-protection subsets for $p \in \{1, 2, 3\}$. Then we generate 100 such random topologies successively and for each such graph the sizes of p-protection

subsets are computed. Finally, we obtain the average size of each of the p-protection subsets of all the 100 topologies. The value of n is then incremented by 100 (i.e., $n = 200$) and the whole procedure is repeated. We continue incrementing the value of n by 100 each time and repeating the experiment until $n = 500$.

The results are plotted in Figure 7.4, where the x-axis shows the number of nodes in the generated topology and the y-axis represents the average sizes of p-protection subsets (average sizes of the 1-protection subsets are shown by the solid line, 2-protection by the dashed line and 3-protection by the dotted line). For the 1-protection subset, the average sizes remain more or less stable for different values of n. As can be seen from the graph, average sizes for 2 and 3-protection subsets increase for larger values of n. In the following, we explain the behavior of the curves. The reason of the increase of sizes of 2 and 3-protection subsets is given below.

Given a constant transmission radius for the nodes and fixed field size, for smaller values of n (100, or 200) a node in a random topology has a smaller size neighborhood than for larger values of n (400 or more). Thus in the case of smaller values of n, where the neighborhood of a node is smaller, the probability that two or more nearby MIS nodes select the same node is relatively high. Similarly for larger values of n, when the neighborhood of a node increases the probability that two or more nearby MIS nodes select the same nodes is lower because there are lots of other nodes to choose from (a node's neighborhood). That is why, the number of nodes selected by the MIS nodes increases for higher values of n. However, we see a sharp rise in the number of active nodes constituting 2 and 3-protection subsets than that of 1-protection subset. This is because the number of nodes selected by an MIS node in 2 and 3-protection subsets is at most 15 and 20, respectively (which is exactly 1 for

the 1-protection subset).

In Figure 7.5, we show the average approximation ratios generated for p-protection subsets for different values of n. We determine the approximation ratios by comparing the sizes of p-protection subsets generated by the algorithm with the lower bound on the corresponding sizes by the optimal algorithm (given in Theorem 7.4.2). Following the argument given in the previous paragraph, we find that the approximation ratios for the 1-protection subset remains stable around 7 for all values of n, whereas the approximation ratios for 2- and 3-protection subsets increase for larger values of n.

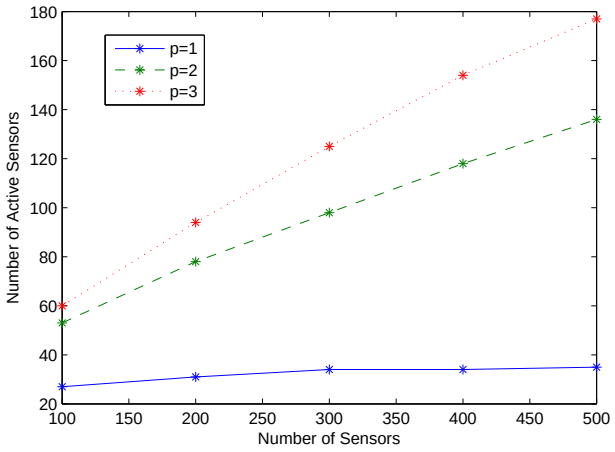


Figure 7.4: Showing average sizes of p-protection subsets for different values of $n \in \{100, 200, \cdots, 500\}$ and $p \in \{1, 2, 3\}$.

As before, we compute standard deviations of the sizes of p-protection subsets ($p \in \{1, 2, 3\}$) to assess the performance of the algorithm. Figure 7.6 shows the standard deviations of p-protection subsets. It can be observed that the data for p-protection subsets tend not to vary a lot from the mean. However, we see a little more variation in the obtained values for $p = 2, 3$ than $p = 1$. This is because for

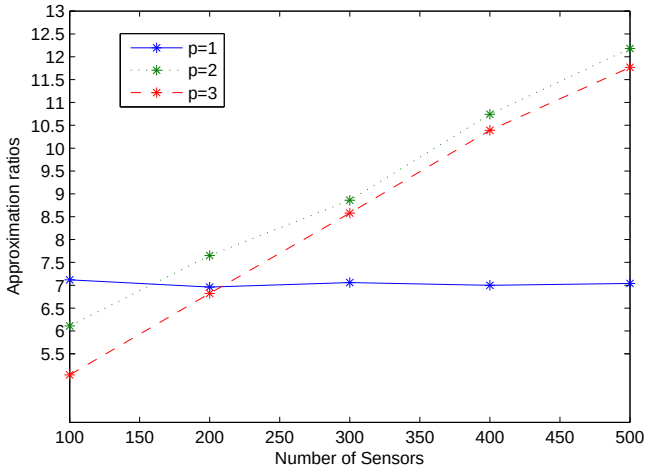


Figure 7.5: Showing average approximation ratios for p-protection subsets for different values of n.

larger p's the number of neighbors selected by the MIS nodes increases which, in turn, increases the probability of obtaining a varying number of selected nodes for different randomly generated graphs. Furthermore, with larger n this probability increases even more which results in swelling the standard deviations.

7.6 Conclusion

Providing protection to sensors by their neighbors in a WSN is an interesting problem. The need to protect sensors arises because they are vulnerable to attacks, failure or damage while they monitor objects or provide coverage to a certain area. By having some *protector* sensors around others allows us to know the status of these sensors and take appropriate actions (replacing the dying and faulty sensors, if possible) in case those *sensing* sensors fail or run out energy. In this chapter, we studied the problem

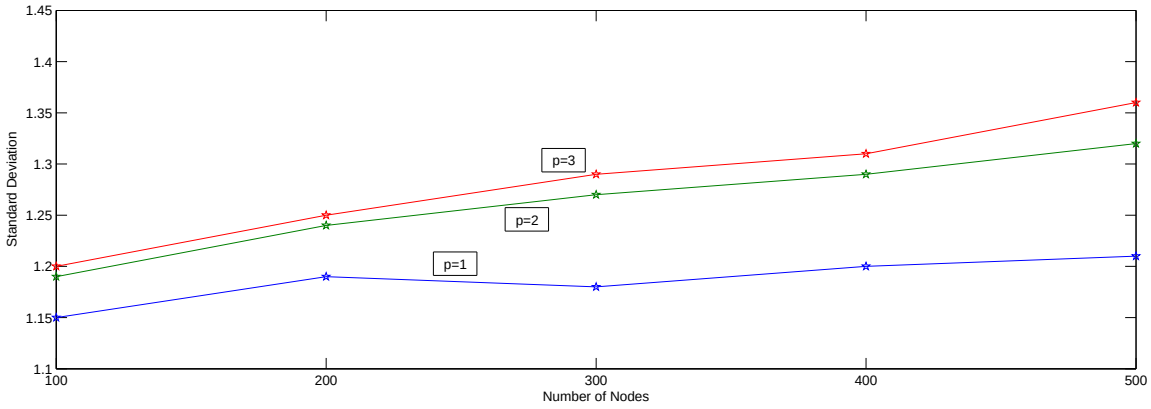


Figure 7.6: Showing standard deviations of the sizes of p-protection subsets for different values of n.

of protecting the sensors by one another namely, the p-protection subset problem, and provide a distributed solution with a constant approximation factor. To the best of our knowledge, this is the first constant factor approximation ratio to the p-self-protection subset problem which is significantly better than the previous $O(\log n)$ approximation ratio. We conducted extensive simulation to assess our algorithm and show that the algorithm produces better results than the theoretical bounds established.

We have identified some open problems that result from this work. Our constant factor approximation is relatively large because we focus only on the worst case behavior, however we believe that the approximation ratio will be much smaller in general cases. Besides, the p-protection subset we find is not necessarily connected, thus, finding a connected p-protection set would be interesting. Another challenging open problem is to find a disjoint family of p-protection sets such that each such set can be used for a certain period of time and then can be replaced iteratively by other sets in order to balance energy consumption.

Chapter 8

Topology Control

In previous chapters we mainly focus on finding subsets of nodes from the underlying networks that are useful for different applications of WSNs. Here we are interested in finding subsets of edges (instead of nodes) which allow various problems on WSNs to be solved efficiently. Generally, topology control plays an important role in the design of efficient algorithms for applications such as coverage, connectivity and routing in WSNs [44, 45, 48, 75, 84, 50, 49, 63, 65, 64, 80, 81, 19, 32, 6]. The main idea is to obtain a subgraph from the underlying network graph by removing some (possibly redundant) edges in order to enhance the overall performance of the network in some *ways*.

We present a local topology control algorithm (LTCA) in WSNs which has certain desirable properties. First, the algorithm is very simple, strictly local (each node requires only 1-hop information of its neighbors) and fast (requiring each node to communicate with its neighbors exactly twice). Second, LTCA does not assume the underlying graph to be a UDG, and in fact works on general connected graphs. Third and the most important feature of LTCA is that it is completely independent

of any location information of nodes; it relies only on the connectivity information and the identities (ids) of the neighboring nodes. Assuming nodes are deployed in the plane, it is shown that the resulting subgraph obtained by our algorithm is connected, symmetric and contains few edge crossings. We provide simulation results which show that on random graphs the resulting topology is a *spanner* and the average degree of the subgraph is *low*. Due to the simplicity of the algorithm, LTCA can easily be implemented in practical WSNs.

8.1 Introduction

In order to reduce the power consumption of the network and extend its lifetime, topology control algorithms, in general, deal with finding a suitable structure (strictly speaking, a spanning subgraph) of the given graph. This resulting subgraph is expected to have certain features (e.g., connectedness, planarity, sparseness, bounded-degree, etc.) which facilitate, among other things, routing in the network. Given a UDG $G = (V, E)$, topology control algorithms focus on constructing a subgraph $G' = (V, E_{TC}), E_{TC} \subseteq E$. For transmission, longer links (edges and links are used interchangeably) in the network consume more power than shorter links since the power spent on a link for a transmission is directly proportional to at least the square of the Euclidean length of that link. That is why, the general approach of topology control algorithms is to remove longer links and retain shorter links which is supposed to save energy in the long run for routing and thus help extend the lifetime of the network. However, the disadvantage of this idea is that if too many longer links are eliminated then some paths may be unacceptably longer in terms of the number of hops. This increases the probability of message loss and takes longer to deliver messages. Care

must be taken to design topology control algorithms to meet this challenge.

For any topology control algorithm, the first and foremost requirement is that the subgraph G' be connected. It is expected that G' be symmetric, that is, for any two nodes u and v, u is a neighbor of v if and only if v is a neighbor of u. Among other properties, G' should be planar (a graph is called a *planar* graph if no two edges in G' cross when drawn in the plane) because certain algorithms (for example, GPSR [12]) utilize the planarity property of graphs for successful routing.

Another common requirement is that G' be sparse; that is, the number of edges should be in the order of the number of nodes. Sparsity of a graph helps maintain a simple neighborhood for individual nodes. This is because the smaller the neighborhood, the faster the information processing among the nodes in the neighborhood. One of the important properties of G' is that it should be a spanner. This means that the *cost* of a shortest path between any two nodes in G' must be at most t times the cost of the shortest path between the same two nodes in G. There are a number of cost metrics used in sensor networks. The most often used cost functions are the Euclidean length of links, the Euclidean length raised to a predefined power, etc. The cost of a path is determined to be the sum of the costs of all the links of the path.

The rest of the chapter is organized as follows. In Section 8.2 we review previous work on topology control in WSNs. Section 8.3 provides a few definitions and assumptions. The LTCA is presented in Section 8.4 and an average-case analysis over randomly generated networks is shown in Section 8.5. Conclusions are given in Section 8.6.

8.2 Related work

Topology control in WSNs has been studied extensively, see for example [6, 45, 48, 64, 75, 84, 50, 80], where authors propose centralized schemes as well as distributed algorithms. Since the sensors can be modeled as a set of points in the plane, many algorithms use some of the fundamental results of computational geometry to produce *nice* structures from the underlying graph. These include the minimum spanning tree (MST) [64], the Delaunay triangulation [32], a generalized version of the Gabriel graph (GG) [65], the relative neighborhood graph (RNG) [44], and so on. A Delaunay triangulation-based method to select edges from the underlying graph is described in [32]. Using some heuristics, the algorithm selects edges in such a way that they form a regular and uniform structure and the degrees of nodes are upper-bounded by some constant.

In [64] the authors present a centralized method that constructs a spanning tree from the given graph where the goal is to minimize the maximum power of the network. The maximum power of a sensor is defined as the power required to transmit a message to the furthest neighbor of that sensor. However, they also present two distributed algorithms that adjust the transmission power of individual sensors to reduce the power consumption. The problem with their method is that the algorithms do not ensure connectivity in all cases. However, the algorithms [32, 64] may not be effectively applicable to power constrained WSNs since neither the Delaunay triangulation nor the MST can be computed locally and hence require all the sensors to transmit their positions to the central base station for centralized solutions, which is typically power-consuming.

The GG based solution described in [65] is quite reasonable for WSNs since the

GG can be locally computed and the graph is symmetric and a spanner. The first distributed topology control algorithm which achieves most of the properties described above is called CBTC (Cone-based Topology Control Algorithm) [80]. The subgraph obtained by the algorithm is a spanner, it is planar, sparse and can be distributedly (but not locally) computed. Since then there have been a number of algorithms [40, 46, 76] which have proposed local solutions to obtain these properties.

A locally constructed spanning subgraph known as the low-weighted modified RNG is introduced in [44]. The idea is based on a simple modification of the original RNG where the author shows that the structure obtained by this algorithm is connected and planar. He shows that the sum of all the edge lengths (Euclidean length) of the subgraph is within a constant factor of that of the MST. This method is local, uses only $O(n)$ messages to build such structures and every node uses only its 2-hop neighbor information. Although the total edge length of this structure is within a constant factor of that of the MST, the energy consumption using this structure is not within some constant of the optimum.

Wattenhofer and Zollinger [81] propose a simple topology control algorithm, called the XTC algorithm, that is independent of sensors' specific coordinates in the plane but assumes that approximate Euclidean distance among neighboring sensors are available. As all the previous algorithms utilize the fact that the location of each individual sensor is known, this is the first location-independent algorithm to produce a connected, planar, and degree-bounded topology. XTC also works on general graphs since it is location independent. Although the subgraph is not a spanner, the authors provide simulation results and show that the spanner property holds on average graphs.

The only disadvantage of that algorithm is that a small error in the estimated distance information may generate a disconnected subgraph. That is, exact distance information is crucial for their algorithms to produce a connected structure. Later, in order to circumvent the problem, Kevin and Sriram [52] came up with two randomized solutions (which are a generalized version of [81]) to produce a topology that is guaranteed to be connected. These algorithms are robust to distance error, in the sense that they withstand a certain amount of error in estimating distances between neighbors. Although the resulting subgraph is planar and connected, one of the algorithms cannot guarantee to yield a bounded degree for the resulting topology. In fact, a node in the network topology can have a logarithmic bound on the degree in the original graph. The other randomized algorithm guarantees to produce a subgraph which has bounded degree with high probability.

8.2.1 Failure due to imprecise location information

Here we present instances where two location-based algorithms [81, 65] used for topology control in WSNs can produce disconnected (or a subgraph with edges which are non-existent in the original graph) subgraphs if either there is a small error in the distance information between neighbors [81] or the precise location information is not available [65].

Disconnected Subgraph in [81]

As mentioned before, given a graph the technique in [81] assumes exact distance information between neighbors. The algorithm finds a topology relying only on these distances and the ids of the nodes in the graph. However, the approach suffers from

the problems of disconnectedness and unbounded degree. For a clear exposition, we briefly present their algorithm. The XTC protocol consists of three steps: (i) Neighbor ordering (ii) Neighbor order exchange and (iii) Edge selection. A total order $\prec_u$ is defined and used by each node u to order its neighbors

$v \prec_u w \Leftrightarrow (|uv|, \min\{id(u), id(v)\}, \max\{id(u), id(v)\})$

$< (|uw|, \min\{id(u), id(w)\}, \max\{id(u), id(w)\})$ (1),

where $|uv|$ is the Euclidean distance between u and v. The neighbor orders of the XTC algorithm are based on the lexicographic order of these link weights. Node u drops v from its neighborhood if there exists a w such that $w \prec_u v$ and $w \prec_v u$ (u thus forms an updated neighborhood by removing such v's). Finally, the output of the protocol, that is, the spanning subgraph $G_T = (V, E_T)$ consists of the edge set E_T, where $E_T = \{uv | v \in N(u)\}$. It is shown in [81] that G_T is symmetric, planar and of bounded degree if the underlying graph is connected. The performance depends on the correct distances between the neighbors and it is sensitive to small perturbances in the distances which may produce different orderings.

For illustration, we adopt the example mentioned in [52] which shows that the subgraph produce by the XTC algorithm becomes disconnected with a small error in the distances of the neighbors. For a UDG shown in Figure 8.1 (a), assume $0 < \epsilon < 1 - 1/1\sqrt{2}$ and the lengths of the edges are $|ab| = |dc| = (1 - \epsilon)/2$ and $|ac| = |bd| = 1/2$. The neighborhood orderings $\prec$ according to (1) are:

$d \prec_a b \prec_a c$, $c \prec_b a \prec_b d$, $b \prec_c d \prec_c a$, $a \prec_d c \prec_d b$.

However, if b and c estimate their distances ($|ba|$ and $|cd|$, respectively) incorrectly, i.e., $|ba| = (1+\epsilon)/2$ and $|cd| = (1-\epsilon)/2$, then we obtain the following orderings:

$d \prec_a b \prec_a c$, $c \prec_b d \prec_b a$, $b \prec_c a \prec_c d$, $a \prec_d c \prec_d b$.

With these new orderings, when the XTC algorithm is executed the subgraph becomes disconnected as shown in Figure 8.1 (b). In the same paper [52], the authors also show instances where nodes in G_T can have unbounded degree if some nodes incorrectly estimate distance between them. For details, the reader is referred to [52].

Non-existent edges

We show that due to a small error in the geographic position of a node, the GG can have edges that were not in the original graph. Consider Figure 8.1 (c). Suppose the exact positions of nodes a and b are known and a incorrectly estimates the location of c to be c'. The absolute deviation can be made smaller as it is evident from the figure. Assume edges ab and bc belong to the original graph. According to the definition of the GG, an edge between two nodes a and b exists if the circle with diameter ab does not contain any other node inside the circle. However, due to the inaccurate location information of node c, the edge ab does not exist anymore in the GG, but there will be new edge ac' in GG, which did not originally exist in the input graph. Thus the subgraph contains new edges and may not be used for certain applications.

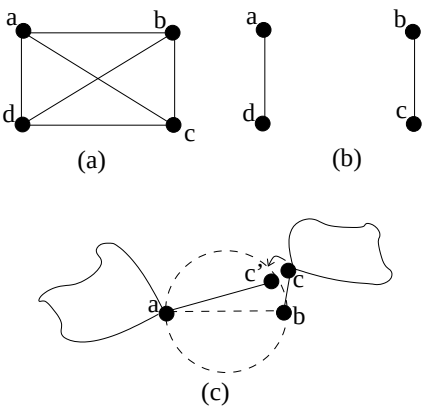


Figure 8.1: The XTC algorithm produces a disconnected graph (b) with a small error in the estimated distances between nodes in the graph shown in (a). (c) If a incorrectly estimates c's actual position then the GG can have new edge ac'.

8.3 Preliminaries

The cost of a link uv in the network is defined to be the amount of power required for sensor u to send a message to sensor v or vice versa assuming symmetric links. The cost of uv is $|uv|$ for the Euclidean metric and $|uv|^\alpha$ for the energy metric, where $\alpha \geq 2$. A path $p_G(s,t) = (s = u_1, u_2, \cdots, t = u_k)$ in G from node s to node t is a sequence of edges $u_i u_{i+1}$ and the cost of $p_G(s,t)$ is the sum of the costs of the edges in $p_G(s,t)$. Let $|p_G(s,t)|$ represent the cost of the optimal path $p_G(s,t)$. A subgraph G' has a *spanner ratio* t if the cost of the optimal path $p_{G'}(s,t)$ is at most t times worse than the optimal cost of $p_G(s,t)$, i.e.,

$$t \geq |p_{G'}(s,t)|/|p_G(s,t)|.$$

Here t is called the Euclidean spanner ratio (resp. the energy-spanner or power

spanner ratio) for the Euclidean metric (resp. the energy metric). We assume all nodes are active and not in the sleep mode during the execution of the algorithm.

8.4 Local topology control algorithm (LTCA)

In this section, we present a deterministic local topology control algorithm ($LTCA$) for WSNs. The assumption is that the nodes are deployed in the two dimensional plane and they do not have any information about their position or distances to their neighbors. We informally describe our algorithm. By transmitting to the maximum transmission power, nodes explore their neighbors, that is, individual nodes learn about the corresponding number of neighbors and their ids. Thus each node u builds its neighbor list $N(u)$.

As the algorithm is executed at each node u, u starts removing some of the neighbors from its neighbor list. Node u checks every pair of nodes $k, m \in N(u)$ to see whether they are direct neighbor to each other. If they are not so or if u has only one neighbor, then u retains its original neighbor list and no removal of neighbors takes place. If they are neighbors to each other, i.e., $km \in E$, then u removes none of its neighbors if $id(u) = \min\{id(k), id(m), id(u)\}$. However, if $km \in E$ and $id(u) \neq \min\{id(k), id(m), id(u)\}$, then u removes from $N(u)$ the node that has the $\max\{id(k), id(m)\}$. Removal of some neighbor k from $N(u)$ means the corresponding link uk is eliminated from the network. Thus u builds an updated neighborlist $N'(u) \subseteq N(u)$.

In Figure 8.2 we give the $LTCA$ algorithm which is executed at each node u. The algorithm appeared in [33].

Algorithm: LTCA
Input: A general connected graph, $G = (V, E)$.
Output: A Spanning subgraph, $G_{TC} = (V, E_{TC} \subseteq E)$.

1. Each node u broadcasts its $id(u)$ to all its neighbors $v \in N(u)$.
2. For each neighbor $v \in N(u)$, u receives its id $id(v)$.
3. For two distinct nodes $k, m \in N(u)$ such that $km \in E$ then remove the node with $\max\{id(k), id(m)\}$ if $id(u) \neq \min\{id(k), id(m), id(u)\}$. Otherwise no neighbors are deleted from $N(u)$. The updated neighbor set of u is denoted as $N'(u) \subseteq N(u)$.

Figure 8.2: The LTCA algorithm.

After the execution of $LTCA$, we find a subgraph $G_{TC} = (V, E_{TC})$, where $E_{TC} = \{uv | \forall u : v \in N'(u), u \in V\}$.

In the following subsection, we provide an analysis of the graph G_{TC} generated by $LTCA$.

8.4.1 Analysis of $LTCA$

As mentioned before, the foremost property of any topology control algorithm is that the resulting graph must be connected, i.e., there must be at least a path between any two nodes in the subgraph. In this section, we show that the resulting graph $G_{TC} = (V, E_{TC})$ generated by $LTCA$ is connected. Unless otherwise mentioned, the given graph $G = (V, E)$ is assumed to be a general graph and not necessarily a UDG.

Theorem 8.4.1 *Given a general graph $G = (V, E)$, two nodes u and v are connected in $G_{TC} = (V, E_{TC})$ if they are connected in G.*

Proof Consider any two distinct nodes u and v in G and a path $\pi = (u = u_1, u_2, u_3, \cdots, u_{t-1}, u_t = v)$ between them. Path π exists since G is connected. If u and v are

also connected by a path in G_{TC} then we are done. Suppose they are not connected in G_{TC}. Then we have two connected components G'_{TC} and G''_{TC} in G_{TC}. Now $\pi = (u = u_1, u_2, u_3, \cdots, u_{t-1}, u_t = v)$ is divided into two subpaths $\pi' = (u = u_1, u_2, u_3, \cdots, u_s)$ in G'_{TC} and $\pi'' = (u_{s+1}, u_{s+2}, \cdots, u_t = v)$ in G''_{TC}, where u_s is the last node in π' in component G'_{TC} and u_{s+1} is the first node in π'' in component G''_{TC} (see Figure 8.3). We analyze the following two cases.

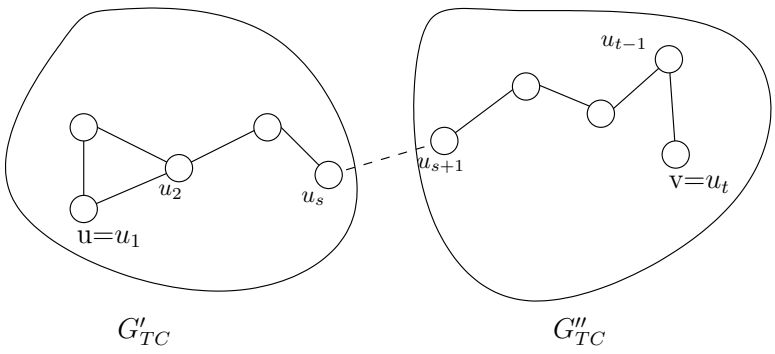


Figure 8.3: Proving that the subgraph obtained by LTCA is connected.

Case 1: $N(u_s) \cap N(u_{s+1}) = \phi$ in G. That is, there is no common node between the neighbors of nodes u_s and u_{s+1}. So, u_s and u_{s+1} form a clique of size two. Without loss of generality, assume $id(u_s) > id(u_{s+1})$. Node u_s will discard u_{s+1} in G_{TC} if and only if there is some node $k \in N(u_s) \cap N(u_{s+1})$ such that $id(u_{s+1}) > id(k)$ (then u_s, u_{s+1} and k form a clique of at least three). This is a contradiction since we assume k does not exist. Therefore, the edge $u_s u_{s+1}$ is retained in G_{TC} which makes G_{TC} connected.

Case 2: $N(u_s) \cap N(u_{s+1}) \neq \phi$ in G. That is, there exists at least some node k such that u_s, u_{s+1}, and k belong to a clique of size at least three. Consider such a clique C of size p (u_s, u_{s+1}, and k belong to C). In C, we can have at most $p - 2$

cliques of three nodes, where u_s and u_{s+1} are fixed.

According to the algorithm, there are two ways u_s and u_{s+1} cannot be neighbors to each other in G_{TC}, namely $id(u_s) \neq \min\{id(m) : m \in C \setminus \{u_s\}\}$ or $id(u_{s+1}) \neq \min\{id(m) : m \in C \setminus \{u_{s+1}\}\}$. This is because if either u_s or u_{s+1} has the smallest id in one of the $p-2$ cliques then the adjacent edges to the smallest id node are not removed. Thus, if u_s or u_{s+1} has the minimum id in one of the $p-2$ cliques then we are done.

Suppose none of them has the minimum id. Then it is easy to see that we could reach u_{s+1} from u_s via the minimum id node in C. This contradicts the fact that G'_{TC} and G''_{TC} are not connected. Therefore, either π' and π'' are connected by edge $u_s u_{s+1}$ or there exists a node $m \in N(u_s) \cap N(u_{s+1})$ (where $id(m) < id(u_s)$ and $id(m) < id(u_{s+1})$) such that u and v are connected via path $\pi' \cup \{u_s u_m\} \cup \{u_m u_{s+1}\} \cap \pi''$. Hence, all pairs of nodes are connected in G_{TC}. ∎

In the following, we prove that the edges in G_{TC} are symmetric, that is, if node u includes v as its neighbor in G_{TC}, then v includes $u \in N'(v)$. Having the subgraph symmetric is an important characteristic, because asymmetric communication graphs are generally hard to realize in practical situations as investigated in [63].

Theorem 8.4.2 *Given a general graph $G = (V, E)$ (with $uv \in E$), node u includes v in $N'(u)$ if and only if v includes u in $N'(v)$ in G_{TC}.*

Proof We analyze two cases.

Case 1: $N(u) \cap N(v) = \phi$. Then according to the last step of LTCA, both u and v includes the other in their respective updated neighbor list. Therefore, if u includes v in $N'(u)$ then so does v.

Case 2: $N(u) \cap N(v) \neq \phi$. Consider a common neighbor $w \in N(u) \cap N(v)$. Assume u includes v in $N'(u)$. We now show that v also keeps u in $N'(v)$. Since $v \in N'(u)$, we have either $id(v) < id(w)$ and $id(u) \neq \min\{id(w), id(v), id(u)\}$ or $id(u) = \min\{id(w), id(v), id(u)\}$ (the relation of ids between v and w does not matter). For the former situation, if $id(v) = \min\{id(w), id(v), id(u)\}$ then u is retained in $N'(v)$ (step (iii) of the algorithm) and for the latter situation v includes u and discards w since $id(w) > id(u)$.

To prove the other direction, i.e., v's inclusion of u in $N'(v)$ implies $v \in N'(u)$, simply exchange the role of u and v in the previous paragraph and the proof follows. ∎

Now we focus on the planar property of G_{TC} since achieving planarity in the resulting subgraph is one of the expected features in topology control algorithms. The algorithm does not guarantee that G_{TC} be always planar. However, it can remove a substantial number of intersecting edges. In the following lemma, we show that all the intersecting edges of a clique C ($|C| > 3$) in G can be removed in G_{TC}. This may greatly reduce the number of intersections in G_{TC}.

Lemma 8.4.3 *Let C ($|C| > 3$) be a clique in G. All the intersections between edges wx and yz, where $w, x, y, z \in C$ can be removed in G_{TC}.*

Proof Without loss of generality, let u be the smallest id node in C. According to *LTCA* (step (iii)), for any two nodes $w, x \in C$, w and x both will discard x and w from their respective updated neighbor lists $N'(w)$ and $N'(x)$ (since $id(w), id(x) > id(u)$). As a result, the edge wx is eliminated from G_{TC}. As all edges $wx \in E$ (where $w, x \in C$ and $u \neq w, x$) are removed, there exist only edges uv, $v \in C \setminus \{u\}$. Therefore, all the intersections between edges wx and yz, where $w, x, y, z \in C$ are removed in G_{TC}. ∎

However, some edge $cd \in E$, where $c \in C \setminus \{u\}$ and $d \in V \setminus C$ can intersect some edge ab, where $a, b \in C$. Elimination of such intersections is not possible (with only connectivity information and no geographic location information) without disconnecting the graph. Situations like the ones shown in Figure 8.4, where the graphs 8.4(b) and 8.4(c) are indistinguishable from each other (labels indicate the corresponding ids of the nodes) and hence intersections cannot be removed with only connectivity information. However, as illustrated in Figure 8.4(a) (right) we can obtain a planar subgraph from the clique shown of Figure 8.4(a)(left) assuming u has the lowest id in the clique.

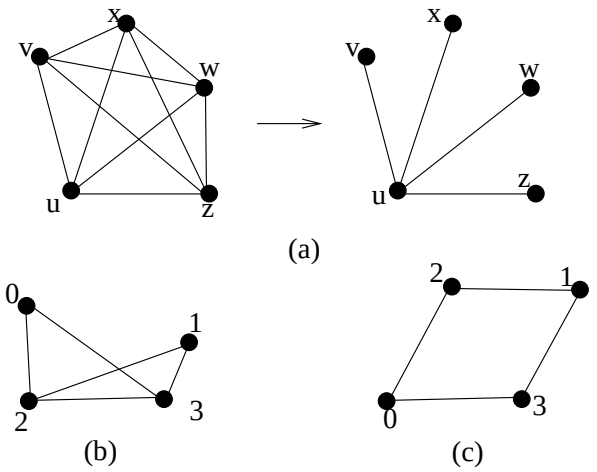


Figure 8.4: Situations like the ones shown in (b) and (c) are indistinguishable from each other and hence intersections cannot be removed with only connectivity information: labels indicate the corresponding ids of the nodes. As illustrated in (a) (right), we can obtain a planar subgraph from a clique (left), assuming u has the lowest id in the clique.

Theorem 8.4.4 *Given a geometric graph $G = (V, E)$, any cycle in the subgraph $G_{TC} = (V, E_{TC})$ generated by LTCA has length at least 4.*

Proof The proof can be derived from Lemma 8.4.3. ∎

8.5 Analysis for random graphs

In this section, we study and provide experimental results regarding the sparseness and spanner properties of the topology G_{TC}. For simulations, we assume that the underlying graph is a UDG in which the spanner and sparseness properties of G_{TC} are analyzed. For this purpose, we generate UDGs by placing nodes uniformly and randomly on a given fixed square and demonstrate the spanner property of G_{TC} for these graphs. For the sparseness property of G_{TC}, we show that the average degree of the nodes of this graph is low for each randomly generated UDG, although the maximum degree of G_{TC} is not bounded. This low degree indicates that G_{TC} is sparse in the average-case. In Figure 8.5, we show the UDG, the Gabriel graph (GG) and G_{TC} of 600 nodes placed randomly and uniformly, where the minimum distance between any two nodes is at least 0.4 unit.

To evaluate the different properties of G_{TC}, we compare it with the GG. Recall that an edge uv between two nodes u and v in the GG exists if the circle with uv as the diameter does not contain any other nodes inside it. We choose the GG since it is one of the most prominent topology control structures which is connected, planar, energy-spanner and can be locally computed. Hence this is a good candidate for comparison, specially with respect to the energy-spanner property. Although the GG is not a spanner in the Euclidean metric (the cost of an edge uv is the Euclidean distance $|uv|$), it is an optimal energy-spanner when the metric is an energy metric (the cost of an edge uv is some power of distance $|uv|^{\alpha}$, where α is in between 2 and 6).

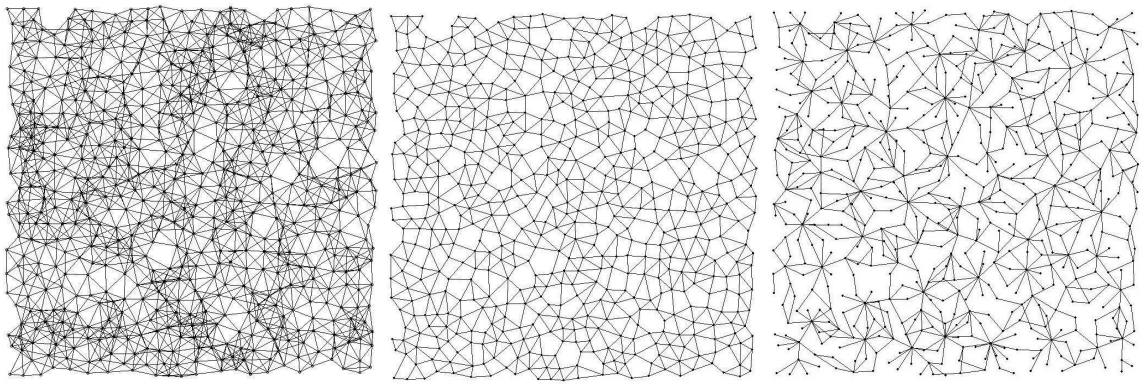


Figure 8.5: The UDG (left), the Gabriel graph (center) and G_{TC} of 600 nodes placed randomly and uniformly are shown, where the minimum distance between any two nodes is at least 0.4 unit.

8.5.1 Spanner and Sparseness properties of G_{TC}

Before we talk about the spanner and sparseness properties of G_{TC}, first we define an important parameter called the network density. Network density [19] is defined as the number of nodes per unit disk. It is shown in [19] that this parameter influences the spanner property of UDGs. We consider a number of different network density values starting from a low density, where there are 4 nodes/unit disk, 6 nodes/unit disk, 12 nodes/unit disk, 25 nodes/unit disk. We consider both the Euclidean and energy-spanner with these different network densities for the GG and G_{TC}. Recall that G_{TC} has a spanner ratio t when

$$t \geq |p_{G_{TC}}(s,t)|/|p_G(s,t)|.$$

For simulations, we consider that the cost of an edge uv is $|uv|$ and $|uv|^2$ for the Euclidean and energy metric, respectively. For each network density, we randomly generate 1000 different UDGs of the same size and for each such graph randomly choose a pair of nodes to compute the Euclidean and energy-spanner ratios in both the GG and G_{TC}. Then we compute the maximum and the average spanner ratios of

all the 1000 graphs for each network density. Figure 8.6(left) depicts the results of the spanner ratios of the GG and G_{TC} with respect to the Euclidean metric. The solid and the dashed lines represent the spanner ratios of the GG and G_{TC}, respectively. Mean values are plotted in black and max values in gray. It can be seen from the figure that the GG has a mean spanner ratio which is slightly above 1.1 and the maximum value is around 1.35. In the same Euclidean metric, G_{TC} has a steady mean around 1.2. However, the maximum is less stable than that of the GG but remains below 7. We observe that the lower the network density the better the spanner ratio in G_{TC}.

The energy-spanner ratio is shown in Figure 8.6 (right), where the solid and dashed lines denote the energy-spanner ratios for GG and G_{TC}, respectively. Since the GG contains an energy-minimal path between any pair of nodes its energy-spanner ratio is exactly one (mean and max values are the same). G_{TC} has a maximum energy-spanner ratio as high as 13, but it maintains an average energy-spanner ratio below 1.5 in all considered network densities. In total we ran our simulations on 1000 randomly generated graphs with different density values that produced the above performance.

We also perform simulations to study the average-case behavior of node degree in both the GG and G_{TC}. Figure 8.7 shows that the G_{TC} has a better average degree than that of the GG. For each of the random graphs with a different number of nodes, the average degree of the GG is always high (at least 3.5), whereas the average degree of G_{TC} is always below 3.4. Although they are not stable, the gap between them is significant.

As spanner ratios play an important role in topology control, we wish to see how our algorithm performs in terms of spanner ratios. Although we plot mean and max spanner ratios in the energy and Euclidean metrics, they do not reveal the

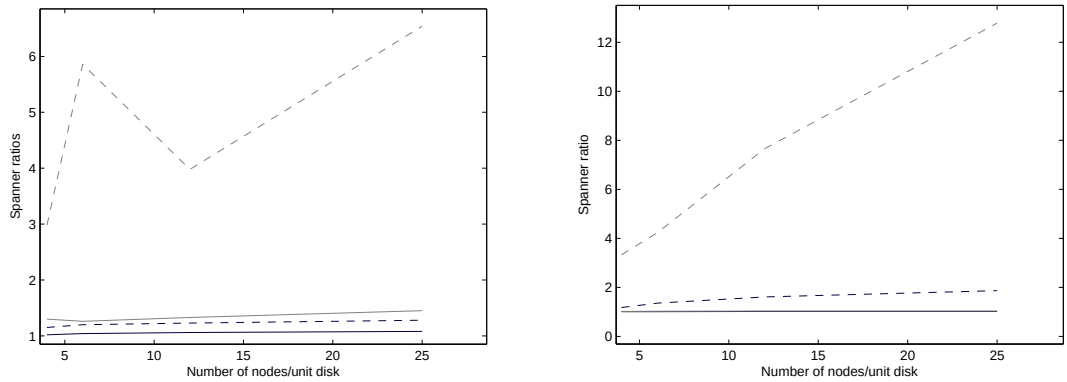


Figure 8.6: Spanner ratios of the GG and G_{TC} w.r.t the Euclidean metric (left). The solid (resp. the dashed) line represents the spanner ratio of GG (resp. G_{TC}). Mean values are plotted in black and max values in gray. Spanner ratios of the GG and G_{TC} w.r.t the energy metric is shown (right). The solid (resp. the dashed) line represents the energy-spanner ratio of the GG (resp. G_{TC}). Mean values are plotted in black and max values in gray. Since the GG contains an energy-minimal path between any pair of nodes its energy-spanner ratio is one (solid line). So its max and mean values coincide.

actual performance of the algorithm. In Figure 8.8, we show standard deviations of the spanner ratios in both metrics. As the network density increases, the standard deviations of spanner-ratios for both metrics increase significantly. This is because in dense graphs, generated by high network densities, the algorithm can remove 'good' edges which are on the optimal path between two nodes s and v in G. Removing such edges in G_{TC} can result in paths with cost $|p_{G_{TC}}(s,t)|$ which can be arbitrarily worse than $|p_G(s,t)|$. Since each node has only minimal information about its neighbors (only neighbors' ids) and no knowledge about the distance of their neighbors, it is not hard to come up with an instance that makes the algorithm perform poorly.

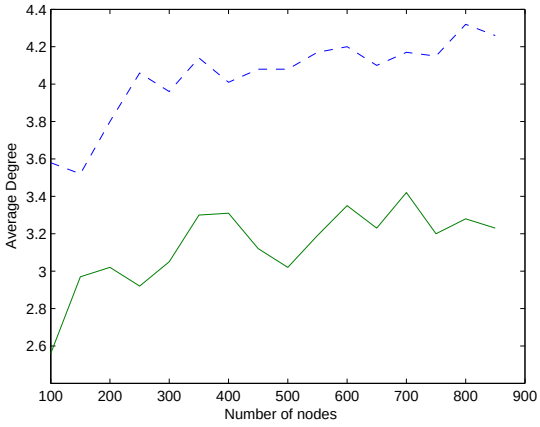


Figure 8.7: Average degree of the GG and G_{TC} in random graphs of different sizes. The dashed and the solid lines denote the sparseness of the GG and G_{TC} respectively. As can be seen from the figure, the average degree of G_{TC} is always smaller than that of the GG in all the instances of different sizes of graphs.

8.6 Conclusion

In this chapter, we presented a simple algorithm $LTCA$ for topology control in wireless sensor networks which has certain useful properties. This one-hop local algorithm is very simple, requires only the ids of the neighbors and assumes no geometric information at all. Each node communicates with its neighbors only twice for sending and receiving ids and the algorithm terminates after two rounds. Unlike most other techniques, $LTCA$ does not assume the graph to be a unit disk graph. In fact it works for all connected graphs since it does not use any geometric information. One fundamental property is that it always produces a connected spanning subgraph from the underlying graph. This property is robust because many existing well-known topology control protocols (such as the Gabriel graph, the Relative neighborhood graph,

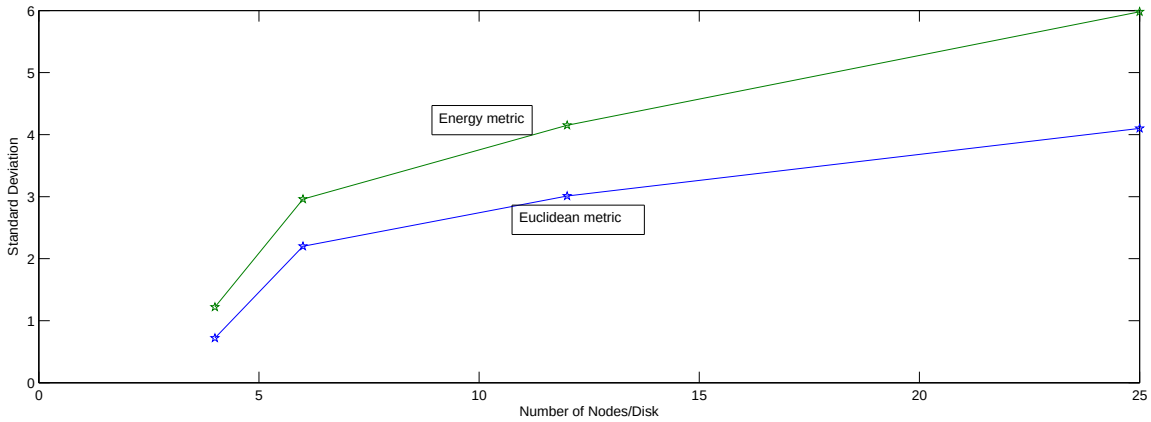


Figure 8.8: Showing standard deviations of spanner ratios of the algorithm for the Euclidean and energy metrics.

the XTC-algorithm) fail to produce a connected subgraph if the exact location information of the nodes is not known or there are errors in the estimation of distances between nodes.

In practical situations, providing precise location information may be expensive and even challenging and a slight error in the geographic location of nodes results in a disconnected subgraph. We show that the subgraph generated by $LTCA$ is symmetric and has few intersections. We provide extensive simulation results for the sparseness and spanner property in random network graphs. Comparing with the well-known topology, the Gabriel graph, it is shown that we can obtain good results in terms of sparseness and spanner ratios in average graphs.

Chapter 9

Conclusion and Future Works

In this thesis, we investigated six different problems in the areas of broadcasting, routing, data collection, target monitoring, and topology control in WSNs. We proposed a number of novel techniques to address these problems and obtained some interesting theoretical results which were verified by extensive simulations. However, the research has also generated a number of interesting open problems which I would like to study in depth in my postdoctoral research.

In general, researchers need to model a WSN and make some basic assumptions in order for their algorithms or techniques to work. Often the assumptions made by most algorithms seem to be unrealistic when applied in real-world situations. For example, many 'good' algorithms assume that sensors know their correct geometric coordinates when deployed in an extended area, a WSN can be perfectly modelled as a UDG, etc. In the future, we must start by challenging such 'strong' assumptions and plan to re-investigate some of the 'well-known' problems in order to come up with good solutions. Specifically, we plan to find a technique that does not make such assumptions, to the problem of generating Connected Dominating Sets (CDSs)

which are widely used for routing in WSNs. Finding such a realistic solution would be a significant contribution to research in WSNs since this can be guaranteed to function in practical situations.

Once this is possible, we would focus on the quality of the solution, that is, the size of a CDS. As previously mentioned, the size of a CDS should be as 'small' as possible since smaller-sized CDSs greatly reduce the number of retransmissions and collisions of messages in the network, which eventually helps increase the lifetime of sensors. As the problem of obtaining small-sized CDSs is challenging, we will appeal to advanced combinatorial techniques such as to bound the size of a CDS within a small constant of the optimum solution.

In this thesis, we introduced a technique to generate a number of CDSs instead of just one and show that this is a very promising way to reduce the sensors' energy consumption to a great extent. In future research, we would be interested in analyzing this problem in depth to obtain some results about the size of each of the CDSs found by the algorithm. We believe that bounding the size of each generated CDS could be possible by carefully utilizing the unique structural properties of UDGs and using some fundamental graph theoretic results. Furthermore, we hope to conduct large scale simulations to experimentally evaluate our algorithm.

Another interesting problem called the *Domatic Partition* problem, which asks for finding the maximum number of disjoint dominating sets (DDSs) in UDGs, has a direct connection in increasing the lifetime of a WSN. Although the problem has been studied in [69, 62], no bound was given on the maximum number of DDSs and hence it still remains as an open problem. However, we present extensive simulation results towards solving this problem and based on the convincing simulations together with a

unique property of a UDG namely, a node in a UDG can have at most five independent nodes in its neighborhood), we conjecture that this problem can be solved within a constant factor of the optimum.

To this end, we plan to tackle the problem using computational geometric and graph theoretic techniques such as convex hulls, maximum independent sets. We also plan to pursue a distributed solution to this problem since distributed techniques are preferred in WSNs due to their scalability, low overhead and effectiveness.

In a nutshell, we are interested to actively continue our research in the design and analysis of distributed and local algorithms to the various problems in WSNs. The main objectives are to explore scalable solutions for practical large sensor networks that explicitly deal with enhancing the lieftime and other energy issues of the network and analyze their performance both theoretically and experimentally. We hope this research will yield results of interest to both the research community and to industry.

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Publications

The following lists all publications written during the years of my being PhD student at Queen's University, Canada.

Journal Papers

1. *Target Monitoring in Wireless Sensor Networks: A Localized Approach.* Kamrul Islam and Selim G. Akl. To appear in International journal of Ad Hoc and Sensor Wireless Networks.

2. *Planar Tree Transformation: Positive Results and Counterexample.* Selim Akl, Kamrul Islam, and Henk Meijer. Journal of Information Information Processing Letter (IPL), Elsevier, 2008.

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