Type assignment for intersections and unions in call-by-value languages

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8 April 2003

FOSSACS '03, Warsaw, Poland

Outline

- Motivation
- Language
- Datasort refinements
- Index refinements
- Intersections
- Indefinite types
- Related work
- Conclusion & future work

Motivation

- Conventional type systems: ML, Java, . . .
  - Tractable and easy to use
  - Limited expressive power
- Refined type systems:
  - Tractable and easy to use
  - Better expressive power

- Fully dependent type systems: Nuprl, . . .
  - Undecidable
  - Extremely expressive

Push the envelope

The goal of the Triple Project: create *refined type systems*.

- The present work: Persistent refinements of algebraic datatypes

Type Assignment

- A *type assignment* system *a la* Curry
  - Terms do not contain types
  - Undecidable
- A *bidirectional* system
  - Terms contain some type annotations
  - Two judgments: *inference* $e \uparrow A$, *checking* $e \downarrow A$.
- This paper: Type assignment only

Property Types

- Some types *build* values
  - Example: tuples
- Others talk *about* values
  - Example: parametric polymorphism $\forall\alpha.\ \alpha \to \alpha$
  - $\forall$ describing behavior of the identity fn
- We call the latter *property types*

Property Types

- In our system:
  - $\delta(\mathfrak{i})$: Datasort and index refinement
  - $\wedge$: Intersection: $v : A \wedge B$ means v has type A *and* type B
  - $\top$: Greatest type (0-ary $\wedge$)
  - Π: Universal quantifier over indices (infinitary $\wedge$)
  - $\vee$: Union: $v : A \vee B$ means v has type A *or* type B
  - $\bot$: Empty type (0-ary $\vee$)
  - Σ: Existential quantifier over indices (infinitary $\vee$)

Language

Language + Initial Typing

$$A, B, C, D ::= 1 \mid A \to B$$

$$e ::= x \mid f \mid () \mid \lambda x.\, e \mid e_1(e_2) \mid \mathsf{fix}\ f.\, e$$

$$\frac{B_1 \leq A_1 \quad A_2 \leq B_2}{A_1 \to A_2 \leq B_1 \to B_2}\ (\to) \qquad \frac{}{1 \leq 1}\ (1)$$

$$\frac{\Gamma(x) = A}{\Gamma \vdash x : A}\ (\mathsf{var}) \qquad \frac{\Gamma(f) = A}{\Gamma \vdash f : A}\ (\mathsf{fixvar}) \qquad \frac{\Gamma \vdash e : A \quad A \leq B}{\Gamma \vdash e : B}\ (\mathsf{sub})$$

$$\frac{\Gamma \vdash e_1 : A \to B \quad \Gamma \vdash e_2 : A}{\Gamma \vdash e_1(e_2) : B}\ (\to \mathrm{E}) \qquad \frac{\Gamma, f{:}A \vdash e : A}{\Gamma \vdash \mathsf{fix}\ f.\, e : A}\ (\mathsf{fix})$$

$$\frac{\Gamma, x{:}A \vdash e : B}{\Gamma \vdash \lambda x.\, e : A \to B}\ (\to \mathrm{I}) \qquad \frac{}{\Gamma \vdash () : 1}\ (1\mathrm{I})$$

cbv Semantics

$$\text{Values} \quad v ::= x \mid () \mid \lambda x.\, e$$

$$\text{Evaluation contexts} \quad E ::= [] \mid E(e) \mid v(E)$$

$$\frac{e' \;\mapsto_R\; e''}{E[e'] \;\mapsto\; E[e'']}$$

$$(\lambda x.\, e)\, v \;\mapsto_R\; [v/x]\, e$$

$$\text{fix } f.\, e \;\mapsto_R\; [\text{fix } f.\, e \,/\, f]\, e$$

Key Properties

A practical type system should satisfy

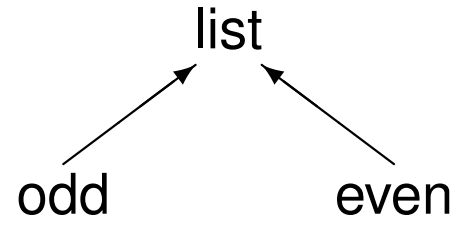
- Preservation: If $\vdash e : A$ and $e \mapsto e'$ then $\vdash e' : A$.
- Progress: If $\vdash e : A$ then e is a value or $\exists e'.\ e \mapsto e'$.

Datasort Refinements

- a.k.a. refinement types (Freeman, Pfenning, Davies)
- Refine an algebraic datatype by a datasort δ
- Example: Lists of integers

$$\begin{array}{ll} \text{Nil} : 1 \longrightarrow \text{list} & \text{Nil} : 1 \longrightarrow \text{even} \\ \text{Cons} : \text{int} * \text{list} \longrightarrow \text{list} & \text{Cons} : (\text{int} * \text{odd} \longrightarrow \text{even}) \\ & \quad \wedge\ (\text{int} * \text{even} \longrightarrow \text{odd}) \\ & \quad \wedge\ (\text{int} * \text{list} \longrightarrow \text{list}) \end{array}$$

δ :



- Intersections essential

Index Refinements

- a.k.a. dependent types restricted to a decidable constraint domain (Xi & Pfenning)
- Refine an algebraic datatype by an index
- Indices drawn from any decidable constraint domain, here $\mathcal{N}$
- Example: Lists indexed by their length

$$\text{Nil} : 1 \to \text{list} \qquad \text{Nil} : 1 \to \text{list}(0)$$

$$\text{Cons} : \text{int} * \text{list} \to \text{list} \qquad \text{Cons} : \Pi a{:}\mathcal{N}.\, \text{int} * \text{list}(a) \to \text{list}(a+1)$$

- Example:
  append : $\Pi a{:}\mathcal{N}.\, \Pi b{:}\mathcal{N}.\, \text{list}(a) * \text{list}(b) \to \text{list}(a+b)$
- Universal quantifier Π essential
- Existential quantifier Σ also essential

Intersection Types

Typing Intersections

- Typing:

$$\frac{\Gamma \vdash e : A_1 \wedge A_2}{\Gamma \vdash e : A_1}\ (\wedge E_1) \qquad \frac{\Gamma \vdash e : A_1 \wedge A_2}{\Gamma \vdash e : A_2}\ (\wedge E_2)$$

$$\frac{\Gamma \vdash e : A_1 \quad \Gamma \vdash e : A_2}{\Gamma \vdash e : A_1 \wedge A_2}\ (\wedge I)$$ 

- $(\wedge I)$ unsound with mutable references.
- $$\frac{\Gamma \vdash v : A_1 \quad \Gamma \vdash v : A_2}{\Gamma \vdash v : A_1 \wedge A_2}\ (\wedge I)$$
- The fix: a *value restriction* [Davies & Pfenning '00].

Indefinite Types

Indefinite Types: Motivation

$$\mathit{filter} : (\mathsf{int} \to \mathsf{bool}) \to \mathsf{list} \to \mathsf{list}$$

$$\mathit{filter} : \Pi a{:}\mathcal{N}.\ (\mathsf{int} \to \mathsf{bool}) \to \mathsf{list}(a) \to \mathsf{list}(\_)$$

$$\mathit{filter} : \Pi a{:}\mathcal{N}.\ (\mathsf{int} \to \mathsf{bool}) \to \mathsf{list}(a) \to (\Sigma b{:}\mathcal{N}.\ \mathsf{list}(b))$$

- $\Sigma b{:}\mathcal{N}.\ B$ quantifies existentially over b in B
- [Xi & Pfenning]

Indefinite Types: Motivation

- Definitely need Σ
- The binary analogue of Σ is the union type $\vee$
- Need $\vee$? Possibly just a convenience
- Issues similar for both, but easier to talk about $\vee$

Union Types

- Introduction rules straightforward:

$$\frac{\Gamma \vdash e : A}{\Gamma \vdash e : A \vee B}\ (\vee I_1) \qquad \frac{\Gamma \vdash e : B}{\Gamma \vdash e : A \vee B}\ (\vee I_2)$$

- Elimination rule?

Rough Road

- Recall: for $\wedge$, introduction rule $(\wedge\mathrm{I})$ was tricky
- Duality suggests $(\vee\mathrm{E})$ will be interesting too

$\vee$ Elimination

- Want to reason by cases: For $x : A \vee B$, show for $x : A$ and for $x : B$.
- Substitution approach (e.g. [MacQueen et al.'86]), reminiscent of natural deduction:

$$\frac{\Gamma \vdash e' : A \vee B \qquad \Gamma, x{:}A \vdash e : C \qquad \Gamma, x{:}B \vdash e : C}{\Gamma \vdash [e'/x]\, e : C}$$

- Suppose $\Gamma \vdash h : (A \to C) \wedge (B \to C)$, $\Gamma \vdash v : A \vee B$

$$\frac{\Gamma \vdash v : A \vee B \qquad \Gamma, x{:}A \vdash h\, x : C \qquad \Gamma, x{:}B \vdash h\, x : C}{\Gamma \vdash h\, v : C}$$

$\vee$ Elimination

$$\frac{\Gamma \vdash e' : A \vee B \qquad \begin{array}{c}\Gamma, x{:}A \vdash e : C \\ \Gamma, x{:}B \vdash e : C\end{array}}{\Gamma \vdash [e'/x]\, e : C}$$

- Consider a nondeterministic choice $e_1 \oplus e_2$:

$$e_1 \oplus e_2 \mapsto e_1 \qquad e_1 \oplus e_2 \mapsto e_2$$

- Suppose $\Gamma \vdash g : (A \to A \to C) \wedge (B \to B \to C)$, $\Gamma \vdash y : A$, $\Gamma \vdash z : B$.

$$\frac{\Gamma \vdash y \oplus z : A \vee B \qquad \Gamma, x{:}A \vdash g\, x\, x : C \qquad \Gamma, x{:}B \vdash g\, x\, x : C}{\Gamma \vdash g\, (y \oplus z)\, (y \oplus z) : C}$$

- $g\, (y \oplus z)\, (y \oplus z) \mapsto g\, y\, (y \oplus z) \mapsto g\, y\, z$
- Unsound whenever a term evaluates to more than one value

$\vee$ Elimination

$$\frac{\Gamma \vdash e' : A \vee B \qquad \begin{array}{c}\Gamma, x{:}A \vdash e : C \\ \Gamma, x{:}B \vdash e : C\end{array}}{\Gamma \vdash [e'/x]\, e : C}$$

- Restrict to exactly one occurrence of e'?
  - Can fail if e' inside a λ (see paper)
- Restrict e' to a value? [van Bakel '99]
  - Sound—but still must guess occurrences of e'

$\vee$ Elimination: Our Solution

$$\frac{\Gamma \vdash e' : A \vee B \quad \begin{array}{c}\Gamma, x{:}A \vdash E[x] : C \\ \Gamma, x{:}B \vdash E[x] : C\end{array}}{\Gamma \vdash E[e'] : C}\ (\vee E)$$

- Exactly one e', *in evaluation position*
- So e' is the next thing to be evaluated
- Remainder of computation (E) on the result of e'

The Empty Type $\bot$

- $\bot$ is the empty or void type

$$\frac{}{\bot \leq A}\ (\bot\mathsf{L})$$

- Elimination rule: by analogy with $(\vee\mathsf{E})$

$$\frac{\Gamma \vdash e' : \bot}{\Gamma \vdash \mathsf{E}[e'] : \mathsf{C}}\ (\bot\mathsf{E})$$

A Generalization?

- Must we restrict the rules to an evaluation context?
- Or could we also allow cases where e' must be evaluated exactly once but might not be in evaluation position?

$$\frac{\Gamma \vdash e' : \bot \quad e' \text{ eval'd exactly once in } e}{\Gamma \vdash e : C}$$

- . . . let $\omega = (\text{fix } f.\ f)$. Then $\omega : \bot$.

$$\frac{\Gamma \vdash \omega : \bot}{\Gamma \vdash (\ (\ )\ (\ )\ )\omega : C}$$

- But $(\ )\ (\ )$ is not a value and does not step

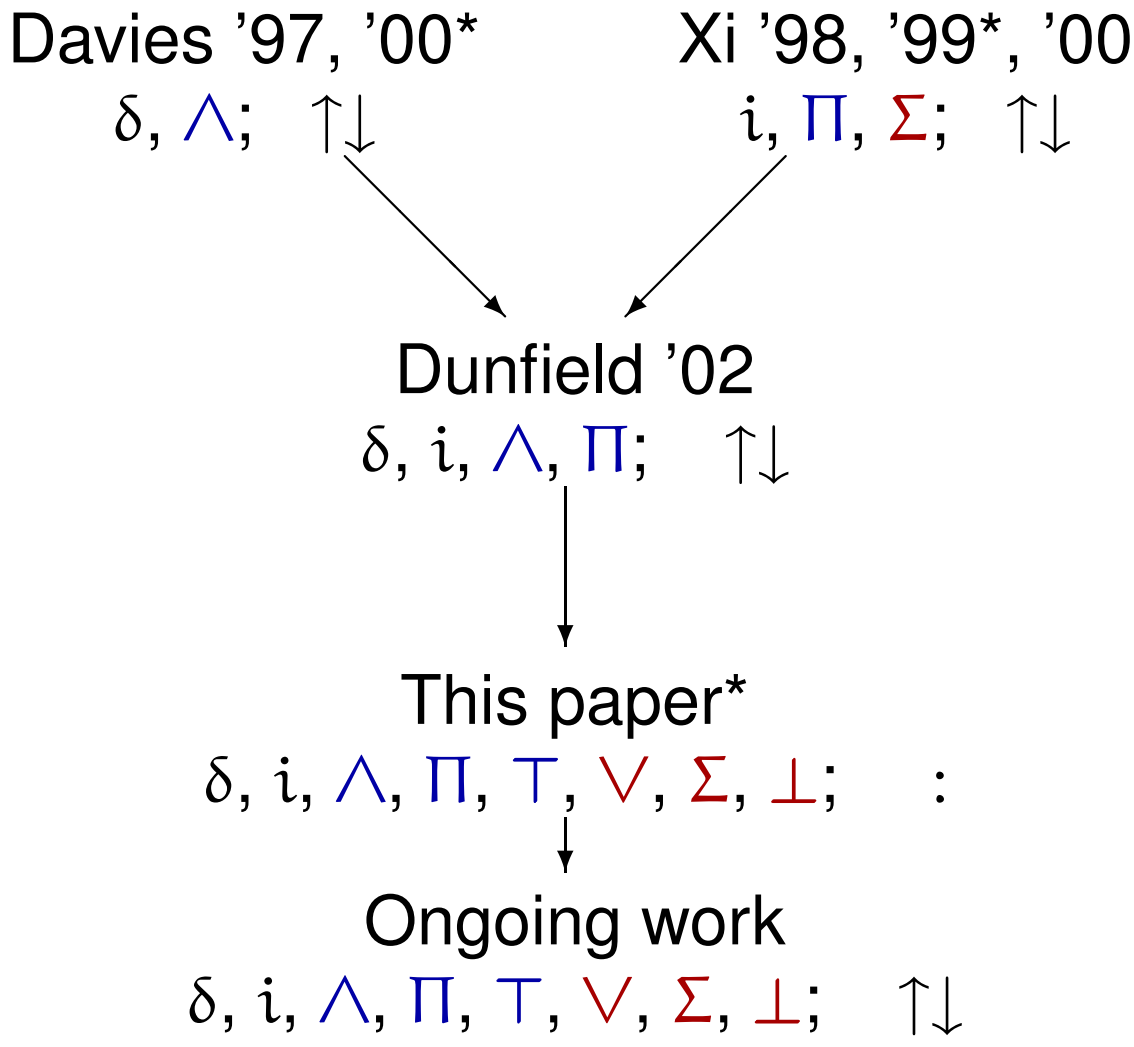
Type Safety

Type Safety

$$\frac{\Gamma \vdash e' : A \vee B \qquad \begin{array}{c}\Gamma, x{:}A \vdash E[x] : C \\ \Gamma, x{:}B \vdash E[x] : C\end{array}}{\Gamma \vdash E[e'] : C}\ (\vee E) \qquad \checkmark$$

- Preservation and progress hold with $\delta, \imath, \wedge, \top, \Pi, \vee, \bot, \Sigma$
- Lemma: Values always have definite types.
  - $\not\vdash v : \bot$
  - If $\vdash v : A \vee B$ then $\vdash v : A$ or $\vdash v : B$
  - If $\vdash v : \Sigma a{:}\mathcal{N}.\, A$ then $\vdash v : [i/a]\, A$ for some i
- Above: a special case of the actual lemma
- (Fully written out for $\delta, \imath, \wedge, \Pi$ in [Dunfield '02])

Closely Related Work



* & Pfenning

Related Work

- [Coppo et al.'81]: $\wedge$ can characterize normal forms (termination); hence undecidable
- [Reynolds '96]: FORSYTHE with $\wedge$ (& type annotations)
- [Pierce '91]: Language with $\wedge$, $\vee$, syntactic markers
- [Barbanera et al.'95]: With unrestricted $\vee$-elim rule, types not preserved under β-reduction
- [Wells et al.'02], [Palsberg & Pavlopoulou '01]: $\wedge$, $\vee$ for control flow analysis

Conclusion

Summary

- Goal: express more invariants
- Two especially useful refinements already developed:
  - Datasort refinements
  - Index refinements
- Refinements motivate $\wedge$, $\vee$, Π, Σ
- $(\wedge \mathrm{I})$: Value restriction
- $(\vee \mathrm{E})$: Eval. context restriction
- Type safety
- The present system: Undecidable, hence impractical

Future Work

- Bidirectional formulation ✓
- Let-normal form ✓
- Mutable references
- Implementation

Acknowledgments

- *National Science Foundation*: CCR-0204248: Type Refinements; 0121633: ITR/SY+SI: Language Technology for Trustless Software Dissemination; Graduate Research Fellowship
- Feedback on the paper: Brigitte Pientka and the anonymous referees
- Feedback on the talk: Aleksey Kliger, Stephen Magill, Jonathan Moody, Frank Pfenning
- "☠": Henrik Christian Grove, available at a CTAN mirror near you

The End

Intersections: Subtyping

- Subtyping:

$$\frac{A \leq B_1 \quad A \leq B_2}{A \leq B_1 \wedge B_2}\ (\wedge R)$$

$$\frac{A_1 \leq B}{A_1 \wedge A_2 \leq B}\ (\wedge L_1) \qquad \frac{A_2 \leq B}{A_1 \wedge A_2 \leq B}\ (\wedge L_2)$$

- Distributivity?

$$\frac{}{(A \to B) \wedge (A \to B') \leq A \to (B \wedge B')}$$ 

- Again unsound with mutable references. [Davies & Pfenning '00]

Unions: Subtyping

- Subtyping for $\vee$ just the dual of $\wedge$:

$$\frac{A_1 \leq B \quad A_2 \leq B}{A_1 \vee A_2 \leq B}\ (\vee L)$$

$$\frac{A \leq B_1}{A \leq B_1 \vee B_2}\ (\vee R_1) \qquad \frac{A \leq B_2}{A \leq B_1 \vee B_2}\ (\vee R_2)$$

Formulating Refinements

- $P ::= \bot \mid i \doteq j \mid \ldots$

  $\Gamma ::= \cdot \mid \Gamma, x{:}A \mid \Gamma, \alpha{:}\gamma \mid \Gamma, P$

  $e ::= \ldots \mid c(e) \mid \text{case } e \text{ of } ms$

  $v ::= \ldots \mid c(v)$

  $E ::= \ldots \mid c(E) \mid \text{case } E \text{ of } ms$

- Atomic subtyping

$$\frac{\delta_1 \preceq \delta_2 \quad \overline{\Gamma} \vdash i \doteq j}{\Gamma \vdash \delta_1(i) \leq \delta_2(j)}\ (\delta)$$

- $$\frac{\overline{\Gamma} \vdash c : A \to \delta(i) \quad \Gamma \vdash e : A}{\Gamma \vdash c(e) : \delta(i)}\ (\delta I)$$

Formulating Refinements

- Atomic subtyping

$$\frac{\delta_1 \preceq \delta_2 \quad \overline{\Gamma} \vdash i \doteq j}{\Gamma \vdash \delta_1(i) \leq \delta_2(j)}\ (\delta)$$

Evaluation Order

Evaluation Order

- Binary and 0-ary indefinite type elim rules:

$$\frac{\Gamma \vdash e' : A \vee B \quad \begin{array}{c}\Gamma, x{:}A \vdash E[x] : C \\ \Gamma, x{:}B \vdash E[x] : C\end{array}}{\Gamma \vdash E[e'] : C}\ (\vee E) \qquad \frac{\Gamma \vdash e' : \bot}{\Gamma \vdash E[e'] : C}\ (\bot E)$$

- A corresponding *unary* rule?

$$\frac{\Gamma \vdash e' : A \quad \Gamma, x{:}A \vdash E[x] : C}{\Gamma \vdash E[e'] : C}\ \text{(direct)}$$

- Actually a derivable rule: $(\vee E)$ with $A \vee A$
- But not derivable in a bidirectional system

Evaluation Order

$$\frac{\Gamma \vdash e' : A \quad \Gamma, x{:}A \vdash E[x] : C}{\Gamma \vdash E[e'] : C}\ \text{(direct)}$$

- Rule needed so that Π's can be instantiated with existential Σ index variables
- Problem first recognized by Xi
- Formulated a translation to let-normal form
- Added "good" programs, but removed them too
- Current work: a sound, complete let-normal translation